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TESIS DOCTORAL

Firm Size Distribution under Adverse Selection and Weak Property Rights: Consequences for Development

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Introduction

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Improving the productivity, especially for the economies which are lagging behind the industrialized world, is of crucial importance if these economies are to become capable of improving the standard of living for their citizens. Developing countries need to increase their output in order to reduce poverty as foreseen in the Millennium Development Goals that all 193 United Nations member states have agreed, at the Millennium Summit in 2000, to achieve by 2015. Identifying the right policies to increase productivity in these nations has been a great challenge for the economic science. Economic historians typically date modern economic growth as beginning in England about 1780 and slightly later in the United States and continental Europe (Prescott 1998). Until that time, most of the major civilizations have had roughly the same standard of living and the inequalities between nations were just beginning to shape. Nowadays, the difference in output per worker between the most developed countries and the poorest ones have reached an order of thirty-fold[1]. It could be thus argued that finding an answer to what are the causes for the differences in GDP per worker across countries is even more relevant today.

This thesis aims to contribute to this fundamental objective, by providing a theory of how property rights enforcement and the development of an efficient financial system can raise output and improve the standards of living.

As Hall and Jones (1999) argue, the differences in physical capital and educational attainment can only partially explain the variation in output per worker. There is a large variation left in the Total Factor Productivity (TFP), which is the part that cannot be explain by traditionally measured inputs. Measuring TFP requires estimates of the factor inputs, which makes a quantitative analysis of TFP not an easy task. Nonetheless, there is a common opinion among macroeconomists that a theory of TFP is needed in order to explain the different economic evolution among countries (Prescott 1998).

When considering economic development, this thesis is mainly concerned with output levels and not growth rates. Levels capture the differences in long-run economic performance that are most directly relevant to welfare as measured

[1] A worldwide statistical partnership, the International Comparison Program (ICP), collects official data that makes possible to compare the output of economies controlling for differences in price levels. Based on the ICP data, the Penn World Table (PWT) provides additional statistics which are widely used in the literature (see for instance Prescott 1998, Hall and Jones 1999).

by the consumption of goods and services. Easterly, Kremer, Pritchett and Summers (1993) document the relatively low correlation of growth rates across decades. A number of models of idea flows across countries such as Parente and Prescott (1994), Barro and Sala-i-Martin (1995) and Eaton and Kortum (1995) imply that all countries will grow at a common rate in the long run: technology transfer keeps countries from drifting indefinitely far from each other. In these models, long-run differences in levels are the interesting differences to explain, a fact also recognized by some of the cross-country growth literature[2].

This thesis is proposing a theory based on differences in property rights enforcement and on adverse selection. The theory is gradually developed chapter by chapter, as follows.

Chapter one lays down the microeconomic foundations. It describes how capital market imperfections (weak property rights and asymmetric information) can affect the size and productivity of firms and the performance of the financial system. Motivated by the empirical findings in La Porta et al. (1998), the chapter studies how these imperfections affect the functioning of the credit markets and the allocation of funds to firms, in an environment in which the heterogeneous quality of firms and the firm size play a key role in determining productivity. The basic argument is that capital markets deal with a variety of problems that arise from asymmetric information about investment projects between borrower and lender, and these problems are worsened with weak property rights. The bad functioning of financial markets involves inefficient firm size distribution, which in turn affects aggregate productivity and per capita income. The financial intermediaries offer an optimal financial contract to firms. This contract supported in equilibrium sheds light on how property rights quality affects the firm size distribution. Information asymmetries and the heterogeneity in firms' productivity generate the adverse selection problem, which plays a crucial role in this theory.

The research presented in Chapter one is related to the literature on asymmetric information in financial markets[3]. However, firm size in these papers is generally exogenous, and their framework cannot produce predictions about the firm size distribution, which is the main goal of this chapter. At the same time, a rich research on the imperfect enforcement of credit contracts tries to explain the empirical relationships between legal environment and business finance, but, unlike this chapter, does not deal with adverse selection problems[4].

After Chapter one described how the financial system allocates capital to firms in the context of imperfect enforcement and asymmetric information, chapter two is asking the following question: can this capital allocation (which depends on the quality of property rights protection) have significant *aggregate consequences*? And if yes, then how much of the variation in output and productivity observed in the data can be accounted by such a theory? To an-

[2] See for instance Mankiw et al. (1992) and Barro and Sala-i-Martin (1992).

[3] See Stiglitz and Weiss (1981), Bester (1985), Besanko and Thakor (1987), Webb (1991) or Fisman and Krausz (2010).

[4] See, for instance, Allen (1981), Jappelli et al. (2005), DeMarzo and Fishman (2007) or Ellingsen and Kristiansen (2010).

swer these questions, the second chapter proposes a general equilibrium model where households members can either be workers or manage a firm, and credit is needed in order to produce. The functioning of the financial market is affected by the imperfections studied in chapter one: weak property rights and asymmetric information.

The question of aggregate effects of limited enforcement in the framework of financial intermediation has been studied before. Notable contributions include Erosa (2001), Perera-Tallo (2003, 2011), Erosa and Hidalgo (2008) and Amaral and Quintin (2010). The novelty of this chapter's quantitative assessment is that it explicitly takes into account the asymmetric information, one of the main problems affecting credit flows in the economy. The results indicate that aggregate productivity is affected in several ways. First of all, due to lower enforcement, firms are constrained to operate at a non-optimal scale. At the same time, informational imperfections determine a shift of resources from productive to unproductive firms. As the financial sector cannot distinguish firms' individual productivity, it will allocate relatively more capital to the less productive firms than in the case of perfect information. Overall, in the numerical exercises, the model is able to account for significant differences in output between developed economies (such as the United States) and middle-income economies. Moreover, the results suggest that, in addition to improving contract enforcement, reducing informational asymmetries can also contribute to an increase in aggregate productivity. The numerical exercises indicate that reducing informational asymmetries would, on average, further increase output by up to 20%[5].

Although the model in the second Chapter is able to replicate fairly well the differences between middle income and developed countries in output per worker, the generated differences in TFP are small compared to what we see in the data[6]. Given that TFP may be interpreted as an index of technological advancement, we ask the question what stops developing countries to adopt more advanced technologies from developed countries. Chapter two shows that weak property rights can cause a bad allocation of funds to firms, where better (and more innovative) firms receive inefficiently low amounts of credit. If this is the case, then one could inquire whether this ineffective allocation of funds acted as a barrier to the spread of new technologies. One way of modelling this is by introducing a learning-by-doing process[7], where the advancement level of the technologies used by firms in the past increases their ability to use more advanced technology in the present. Thus, if property rights are weak, it could be that the more productive firms accumulate less experience in production than

[5] It should be taken into account that the model only allows to compare economies with or without informational asymmetries (with no intermediate degree allowed). This figure represents the average increase in output if all information about firm productivity in the economy would become public. Hence, it represents an upper bound. Nonetheless, it suggests that reducing informational asymmetries may pay off in the long run.

[6] The model can generate a maximum variation in TFP of 30%. Note that here we refer to the base model.

[7] For other models of learning-by-doing see, among others, Arrow 1962, Baldwin and Krugman 1988, Lucas 1988, 1993; Matsuyama 1992; Stokey 1988; Young 1991, Perera-Tallo 2011.

they would under better enforcement. This could slow down their know-how of using advanced technologies.

Chapter three investigates quantitatively this hypothesis and it turns out it has non-trivial consequences for the technological gap across firms in the economy. In the 'enriched' model, this technological gap is not constant anymore, but varies with the quality of property rights. Thus, a certain difference in property rights protection will feature an increased technological gap between backward and advanced firms, which exacerbates the asymmetric information problems. In turn, this amplifies the total effect of enforcement on the aggregate productivity: while the model with exogenous technology adoption (treated in Chapter two) is able to generate up to 30% variation in TFP, the 'enriched' model´s explanatory power reaches almost 50%.

There are several policy implications for developing nations that can be derived from this work. First of all, the quality of property rights protection may affect in the long run the efficiency of the financial system, the size of firms and ultimately the aggregate productivity. Strengthening property rights may reduce capital market imperfections and contribute to an allocation of capital that would be beneficial for the highly productive and innovative firms. At the same time, policies aimed at reducing informational asymmetries could further improve access to capital for these firms, stimulating in this way the overall efficiency.

How does this fit in the context of previous theories which try to explain differences in output and productivity? The uneven development is a very complex phenomenon which has been approached from various angles. It could be argued that a theory on the differences in productivity may be assessed by its ability to suggest policy measures that can be adopted by lagging countries so as to increase their productivity. However, political economy considerations indicate that the incentives of different social classes to implement these policies deserve also to be considered.

Some theories focus on the institutions and government policies that are favorable to productive activities and encourage capital accumulation, skill acquisition, invention, and technology transfer. For instance, Parente and Prescott (2000) proposed as a candidate the strength of resistance to the adoption of new technologies, which would depend upon the policy arrangement a society employs. Hall and Jones (1999) pointed to corruption of government officials, severe impediments to trade, poor contract enforcement, and government interference in production as relevant obstacles to higher output per worker. This thesis follows a similar line of thought. While the idea that property rights matter for development is not that recent[8], the main contribution of this thesis could be summarized in providing a formal (general equilibrium) modelling for how property rights quality affects output and productivity in the presence of asymmetric information. The proposed model tries to strike the right balance between being simple enough to be easily explained intuitively (and allowing for a closed-form solution), and at the same time capturing some features of the

[8]See for instance Hernando de Soto Polar (1989, 2000).

real world so as to be suitable for a quantitative assessment.

The results of this thesis indicate that both the quality of property rights and the asymmetric information have important consequences for development. Some questions and possible future paths for research could be raised following this analysis. First of all, if property rights have been shown to have significant effects on the output and productivity of nations, then how are these property rights determined and why they vary across countries? Acemoglu et all (2005) provided some examples of how economic institutions, which shape economic outcomes, are determined by political power, which is in turn determined by political institutions and the distribution of resources in society[9]. As their framework is largely verbal rather than mathematical, and thus not fully specified, they identified constructing formal models incorporating and extending their theory to be an important task ahead.

At the same time, the recent economic and financial crisis affecting the world economy since 2008 suggests that reassessing the growing interactions among national economies may also contribute to a better understanding of the phenomena studied in this thesis. One possible way of advancing in this direction could be to draw from theories of international trade and investment[10]. These efforts are left for further research.

[9] See also Acemoglu and Robinson (2012).

[10] See for instance Krugman (1981).

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Chapter 1
Property Rights Protection and Firm Size Distribution under Adverse Selection*

Recent empirical work suggests that firm size and the performance of the financial system are positively related to the effectiveness of law enforcement. This paper provides a theory on how the effectiveness of law enforcement affects the allocation of capital by the financial system to firms, and the distribution of firm size. The framework features asymmetric information, which implies that the financial contract should satisfy incentive constraints. Capital is not efficiently allocated because of two main reasons: i) the firm size, measured as the amount of factor of production capital used by each firm, is smaller than the efficient level (the level in the absence of incentive constraints); ii) the portion of capital allocated by lenders to the most productive firms is also smaller than the efficient level, as they cannot distinguish among borrowers. There are three types of equilibria depending on degree of law enforceability: i) When law enforcement is effective enough, the allocation of capital to firms is efficient. ii) When law enforcement is in a certain middle interval, the more productive firms are underfinanced. iii) Finally, when law enforcement is weaker than a certain threshold, all the firms receive the same amount of capital, which involves an inefficiently small size. Thus, more effective law enforcement eases the incentive constraints, improving the allocation of capital among firms and the firm size distribution, which in turn affects aggregate productivity and per capita production.

1. Introduction

One important research question in economics is why, in poorer countries, resources are directed towards less efficient uses more often than in developed countries. Evidence suggests that less developed economies are characterized by low aggregate productivity and small firms. Thus, Tybout (2000) illustrates that the emphasis on small scale production correlates negatively with per capita income levels across countries. Moreover, Kumar et al. (1999) and Beck et al. (2006, 2008) found that firm size is positively related to financial development and law enforceability. Hall and Jones (1999) and Parente and Prescott (2000), on their part, document important differences in aggregate productivity across countries.

In order to better understand the above observations, this paper proposes a theory of how the effectiveness of law enforcement affects the allocation of capital by the financial system to firms, and the size distribution of firms in a framework of asymmetric information. Our focus on financial-market imperfections is motivated by evidence indicating that financial markets tend to perform badly in poor countries and that productivity is

*Conversations and comments from Antonia Díaz, Marco Celentani and participants in the 4th International IFABS Conference and Carlos III workshop were very helpful.

positively correlated with indicators of financial development across countries (see Beck et al., 2000; Erosa, 2001). We also draw from the empirical findings in La Porta et al. (1998), who have demonstrated that the legal system and its enforcement affects the way financial markets function. The basic argument in our paper is that the financial markets deal with a variety of problems that arise from asymmetric information about investment projects between borrower and lender, and these problems are worsened with imperfect law enforcement.

In our model the capital allocation is done through an optimal financial contract[2] that a financial intermediary (a bank) will offer to entrepreneurs. Entrepreneurs use the credit to fund a project, but they have different productivity (ability) which is known only to them, and this generates an adverse selection problem. To be more precise, there are two types of firms (called good and bad), where the good type is more productive than the bad type. Entrepreneurs can default and, in this case, financial intermediaries can recover only a fraction of their revenue. We interpret this recoverable fraction as the degree of law enforceability of our economy. The imperfect enforcement and the adverse selection imply that the financial contract should satisfy incentive constraints. Firm size is measured as the amount of the factor of production used by the firm: capital[3]. The efficient allocation would occur when firms maximize their profits without any incentive constraint. However, in our framework, capital is not efficiently allocated because of two main reasons. Firstly, firm size is not the optimal one when compared with the efficient allocation. Secondly, as lenders cannot distinguish among borrowers, they allocate an inefficiently large amount of resources to the bad firms when compared with the efficient allocation. In this way, the good firms are underfinanced compared to the efficient allocation.

We study a menu of contracts which maximizes the profits of firms subject to two incentive constraints: i) the non-default constraint, which implies that firms have incentives to repay their debts and ii) the revelation constraint, which implies that firms have incentive to truly reveal their type. We examine how the optimal financial contract offered by financial intermediaries evolves as the degree of law (and contract) enforceability in the economy varies. When the contract enforceability is effective enough (but not necessarily perfect), the incentive constrains are not binding and the allocation of capital to firms is efficient. When the contract enforceability is in a certain middle interval, incentive constraints are not binding for bad firms, but the revelation constraint is binding for good firms, which implies that these have a suboptimal size when they are compared with the efficient allocation, and that the portion of capital devoted to good firms is smaller than the one in the efficient allocation. Finally, when the contract enforceability is weaker than a certain threshold, both good and bad firms are constrained, the first type by the revelation constraint and the second by the non-default constraint. It turns out that in this type of equilibrium both types of firms receive the same amount of capital, which is inefficiently low when compared with the perfect enforcement case.

After studying the equilibrium menu of contracts in partial equilibrium, and once checking that equilibrium exists, we study the contract in a general equilibrium framework. In

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[2]The classic references for optimal financial contracts in imperfect financial markets are, among others, Townsend (1979), Diamond (1984), Gale and Hellwig (1985) and Williamson (1986).

[3]Further details on this measurement of firm size, as well as a discussion of alternative measurements of firm size, are presented in section 2.1.

such an environment, the results about the different intervals of the degree of contract enforceability still hold. However, in the middle interval, bad firms are now overfinanced, while good firms are underfinanced compared with efficient allocation. Furthermore, per capita output and productivity increase with the effectiveness of law enforcement, since more effective contract enforceability eases the incentive constraints. We extend then the framework to allow measuring firm size by the number of employees and we calibrate our model using data for the US manufacturing sector. The results of the numerical exercise suggest that the effect of law enforcement on firm size is significant. We also extend the model to analyze the use by firms of own capital to finance production, and the use of capital remaining after production as collateral.

Thus, we construct a theory that links the effectiveness of contract enforcement with the functioning of the financial market and the firm size distribution, which has implications for the aggregate productivity and the per capita output of the economy. The theory emphasizes the role of asymmetric information problems (adverse selection) in an environment in which the productivity of firms is private information. Our theory may contribute to a better understanding of the empirical findings of Kumar et al. (1999) and Beck et al. (2006, 2008) about firm size being positively related to financial development and law enforceability. In our theory, the degree of law enforcement affects the firm size via the capital allocation by the financial system to firms, with the more productive firms suffering tighter incentive constraints than the less productive firms.

The key message of the paper is that asymmetric information in financial markets may be much more costly (lead to higher inefficiencies) in countries with low enforcement than with high enforcement due to the fact that these frictions afect the firm size distribution and as a consequence agregate productivity. Erosa and Hidalgo (2008) is, to our knowledge, the paper closest to ours. one of the main differences is that they do not focus in firm size distribution, the amount of output per firm in their model is fixed. Furthermore, these authors only assume asymmetric information after contracting. The case of ex-ante asymmetric information, illustrated in seminal papers such as Rothschild and Stiglitz (1976), seems quite relevant. Moreover, as it will become clear from the paper, from a theoretical point of view, modelling ex-ante adverse selection in a general equilibrium is not trivial (as the equilibrium features contracts maximizing each type's payoff separately, not the expected value across types).

In an influential paper, Prescott and Townsend (1984) have provided a general framework that allows the study of Pareto optima and competitive equilibria for economies with adverse selection. They show that equilibrium with adverse selection exist only under certain condition. If efficiency requires cross-subsidies across types, then equilibrium will not exist. The equilibrium in this paper does not feature cross subsidies so that the results in the paper are consistent with the analysis in Prescott and Townsend.

From a broader perspective, our research is related to the literature on asymmetric information in financial markets and to the literature on imperfect enforceability of contracts. Asymmetric information between lenders and borrowers leads to credit rationing in Stiglitz and Weiss (1981). Bester (1985) investigates the conditions under which credit can be rationed in markets with imperfect information. Besanko and Thakor (1987) explore the role of market structure in credit allocation when there is informational asymmetry. Webb (1991) designs long term contracts that can be used in competitive financial markets to separate entrepreneurs of different abilities. Fishman and Krausz (2010) investigate how adverse selection restricts the start off investment of firms in order to borrow at more favorable terms in the future. However, these papers usually feature a fixed firm size. Therefore, firm size in these papers is exogenous, and their framework cannot deliver predictions about the firm size distribution, which is the main goal of this paper. In a related line of literature, when banks face asymmetric information about loan quality, endogenous borrowing constraints will restrict the growth of firm size, like in Albuquerque and Hopenhayn (2004).

Arellano et all (2007) use differences in contract enforcement to explain why firms have different ratios of debt to assets across countries. There is a rich research on the imperfect enforceability of credit contracts which tries to explain the empirical relationships between legal environment and business finance that have been documented in recent papers[4]. The role of imperfect contract enforceability in shaping financial contracts was analyzed, among others, by Allen (1981), DeMarzo and Fishman (2007), Ellingsen and Kristiansen (2010) or Jappelli et al. (2005). However, these former contributions do not deal with adverse selection problems.

The structure of the paper is as follows: in section 2 we present the model and analyze the efficient allocation of capital. In section 3 we examine the equilibrium menu of contracts, while in section 4 we study the effect of the degree of law enforceability on the financial market performance and firm size under partial equilibrium. We prove the existence of equilibrium in section 5. In section 6 we analyze the effect of the degree of law enforceability on the financial market performance and firm size under general equilibrium. We conclude in section 7. Proofs and some technical details are collected in the appendix.

2. The Model

2.1. Technology

There are two types of firms, good and bad, indexed by $j \in \{g, b\}$. The productivity of these firms is indexed by the parameters θ_j, where $\theta_g > \theta_b$. The portion of good firms is ν, being $(1-\nu)$ the portion of bad firms. Firms revenues[5] are given by $\theta_j\, R\,(k_j)$, which depends on the productivity of each firm $\theta_j \in \{\theta_g, \theta_b\}$, and on the amount of capital k_j. $R\,(k)$ is a continuous function on its domain, strictly increasing in both arguments and satisfies: $R\,(0) = 0$, $\lim\limits_{k \to +\infty} R\,(k) = +\infty$, $\lim\limits_{k \to 0} \frac{\partial R(k)}{\partial k} = +\infty$ and $\lim\limits_{k \to +\infty} \frac{\partial R(k)}{\partial k} = 0$.

We measure the firm size of a particular firm by the amount of the factor of production used by the firm: capital[6]. The stock of factor of production capital used by firms has been estimated in the empirical literature studying the productivity at firm level (see for instance Gal, 2013 and the references therein). Nevertheless this does not mean that the results of the paper do not apply to other measurements of firm size. Note that in order

[4]See, for example, Djankov et al. (2008) and the references therein.

[5]In fact the function $\theta_j\, R\,(k_j)$ not only includes the revenues but also the remaining part of capital after production.

[6]Throughout the paper we will refer to the factor of production capital used by the firm as simply the capital of the firm.

to simplify and following the literature on adverse selection and credit markets[7], we have assumed that there is a unique factor of production: capital. However, the result about firm size of the paper can be easily extended for the case in which firm size is measured by the number of workers. Some empirical studies used sales as a measurement of firm size (e.g., Beck et al., 2006). In our theoretical model sales would correspond to the revenues of the firm, which are also monotonically increasing with capital. This means that if we would define firm size by the amount of sales, all the results of the model with regard to firm size would hold.

In order to simplify and following most of the literature, we do not distinguish between revenues in strict sense and the capital that remains after production. We assume also that firms do not have own capital (that is, the capital used by firms, k_j, is entirely financed through a financial intermediary). However, we extend the model to incorporate own capital, and the use of remaining capital after production as collateral, in subsection 7.1.

The contract offered by the financial intermediary to each type j of firms consists of a quantity of capital k_j offered at a price (interest rate) i_j. When the contracts are offered, the financial intermediary knows the revenue function of each type but cannot distinguish the type of each firm. However, each firm is aware of its own productivity, giving rise to an ex-ante asymmetric information problem. For simplicity we assume that both firms and financial intermediaries are risk neutral.

Each firm of type j uses the capital according to its productivity θ_j to generate a revenue (including the remaining part of capital after production) of $\theta_j\, R\,(k_j)$. Law enforceability is imperfect in the sense that the firms either fulfill the contract paying the principal plus the interest rate $(1+i_j)k_j$ or they default: they refuse to return the loan. When a firm defaults, the financial intermediary can take the fraction λ of the revenue of the firm up to its debt, defined as $(1+i_j)k_j$, that is, in case of default the financial intermediary can take from the firm the amount $\min\left\{\lambda\theta_j\, R\,(k_j)\,, (1+i_j)k_j\right\}$. As in Erosa and Hidalgo (2008), we interpret the recoverable fraction in default (λ) as the degree of law (contract) enforceability. When $\lambda = 1$, contracts are perfectly enforceable: the financial intermediary may take the whole revenue of the firm up to its debt if the firm refuses to pay back the loan. When $\lambda = 0$, the financial intermediary cannot appropriate any revenue of the firm if she decides to default.

We assume that when a firm defaults, the financial intermediary incurs bankruptcy costs $\psi\theta_j R(k_j)$.The bankruptcy cost $\psi\theta_j\, R(k_j)$ may be interpreted either as a cost of enforcing the contract when firms default or as an information costs: a cost of "type" verification. In any case, bankruptcy cost will not affect the contract at equilibrium, since we will show that default does not occur at equilibrium. The unique role that plays the bankruptcy cost in the model is to guarantee the existence of equilibrium (see section 5 for details).

Financial intermediaries are perfectly competitive. The financial intermediaries pay the interest rate r to collect funds from depositors. This interest rate will be called from now on the "deposit interest rate". We will assume in sections (2) to (5) that there is partial equilibrium and, consequently, the interest rate paid to depositors, r, is an exogenous

[7]See Stiglitz and Weiss (1981), Bester (1985), Besanko and Thakor (1987), Webb (1991), Fishman and Krausz (2010).

variable.

2.2. Efficient Allocation

We will call *efficient allocation* the allocation that occurs when each firm maximizes its profits without any incentive constraint. This allocation coincides with the one in the equilibrium when law enforcement is perfect ($\lambda = 1$). In this case, there is no default in equilibrium and financial intermediaries charge firms the deposit interest rate without any borrowing limits. The maximization problem of the firm is the typical problem of a competitive firm[8]:

$$\max_{k} \theta_j \, R\,(k) - (1+r)k$$

When there are no incentive problems, firms choose an amount of capital in which the marginal revenue of capital is equal to its marginal cost. We will call the solution of the above problem *optimal capital* or *optimal firm size* of firm type j and we will denoted it by k_j^*:

$$k_j^* = \arg\max_{k} \theta_j \, R\,(k) - (1+r)k \Leftrightarrow \theta_j \frac{\partial \, R\left(k_j^*\right)}{\partial k} = (1+r) \tag{1}$$

Since $R\,(k)$ is strictly concave, $\frac{\partial \, R\left(k_j^*\right)}{\partial k}$ is strictly decreasing, which implies (together with the assumptions that $\lim\limits_{k\to 0} \frac{\partial R(k)}{\partial k} = +\infty$ and $\lim\limits_{k\to +\infty} \frac{\partial R(k)}{\partial k} = 0$) that k_j^* is well defined and unique. Obviously, the optimal size of good firms is larger than the optimal size of the bad firms: $k_g^* > k_b^*$. Furthermore, it follows from the Implicit Function Theorem that the optimal level of capital of the firm is a decreasing function of the interest rate r.

We will take the capital allocation in the efficient allocation as our benchmark. We will call inefficient any other allocation different from the efficient allocation. We will say that a firm has an *inefficiently small size* or that it is *underfinanced*, if her size (measured as the amount of capital) is smaller than its optimal level: $k_j < k_j^*$. We will say that *good firms receive an inefficiently small portion of capital* if the portion of capital which receives such type with respect to all the firms is smaller than the portion that receives in the efficient allocation : $\frac{\nu k_g}{\nu k_g + (1-\nu)k_b} < \frac{\nu k_g^*}{\nu k_g^* + (1-\nu)k_b^*}$. We will say that a *firm size distribution* is *inefficient* if it is different from the one in the efficient allocation.

3. Equilibrium

When a financial intermediary offers a lending contract to a firm, she has two informational problems: first, she does not know whether the firm is going to fulfill the contract or not, and second, she does not know the type of the firm. To overcome these problems,

[8] Alternatively, we could also define the problem of maximization of the firm in the efficient allocation such that firms (the agents) maximizes profits subject to the non-negative profit condition of the financial intermediaries (the principal):

$$\max_{k,i} \theta_j \, R\,(k) - (1+i)k$$

$$s.t. : (1+i)k - (1+r)k \geq 0.$$

Obviously, the result of this problem is the same as the one in the main text.

financial intermediaries should give incentives to firms to fulfill the contract and to reveal their true type. Thus, there are two types of incentive constrains: i) the *non-default constraint*: firms should have incentives to repay the loan; and ii) the *revelation constraint*: firms should have incentives to correctly reveal their type.

The first incentive constraint is the non-default constraint: firms should be better off fulfilling the contact and paying the financial intermediary than defaulting:

$$\theta_j R(k_j) - (1+i_j)k_j \geq \theta_j R(k_j) - \min\{\lambda\theta_j R(k_j), (1+i_j)k_j\} \quad \forall j \in \{g,b\} \tag{2}$$

If a firm defaults, the financial intermediary seizes the fraction λ of her revenue up to its debt, that is the financial intermediary gets $\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$. Thus, the profit of the firm in case of default is equal to $\theta_j R(k_j) - \min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$. If the firm fulfills the contract, she gets her revenue minus the payment to the financial intermediary: $\theta_j R(k) - (1+i_j)k_j$. This incentive constraint means that the firm is better off paying to the financial intermediary, in which case she gets $\theta_j R(k) - (1+i_j)k_j$, than defaulting, in which case she gets $\theta_j R(k_j) - \min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$. Another way to write the non-default constraint (2) is as follows:

$$\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\} \geq (1+i_j)k_j \quad \forall j \in \{g,b\} \tag{3}$$

The above equation means that firms have incentives to fulfill the contract if the payment of firms to financial intermediary in case of default, $\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$, is larger than in case of fulfilling the contract, $(1+i_j)k_j$.

Note that if a firm defaults is because the payment in case of default, $\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$, is smaller than the payment in case of fulfilling the contract $(1+i_j)k_j$ (see non default constraint 3). This implies that if a firm defaults then its payment to the financial intermediary is $\lambda\theta_j R(k_j)$:

$$\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\} = \lambda\theta_j R(k_j)$$

Remark 1 *Since the payment of the firm to the financial intermediary in case of default is always $\lambda\theta_j R(k_j)$, we will substitute from now on the mathematical expression of the payment of the firm to the financial intermediary in case of default* $\min\{\lambda\theta_j R(k_j), (1+i_j)k_j\}$ *by $\lambda\theta_j R(k_j)$.*

Thus, we may rewrite the non default constraint (2) as follows:

$$\theta_j R(k_j) - (1+i_j)k_j \geq (1-\lambda)\theta_j R(k_j) \quad \forall j \in \{g,b\} \tag{4}$$

The second incentive constraint is the revelation constraint: firms should have incentives to reveal their true type:

$$\begin{aligned} &\theta_j R(k_j) - (1+i_j)k_j \geq \max\{\theta_j R(k_{j'})(1-\lambda), \theta_j R(k_{j'}) - (1+i_{j'})k_{j'}\} \\ &\forall j, j' \in \{g,b\} \quad j \neq j' \end{aligned} \tag{5}$$

If, for instance, j is the bad type and j' is the good type, then the above incentive constraint says that the bad type firm should be better off signing the contract destined to her own type (receiving $\theta_j R(k_j) - (1+i_j)k_j$ as a payoff), rather than pretending to be a good type (in which case she gets the maximum payoff between pretending being a good firm and defaulting $\theta_j R(k_{j'})(1-\lambda)$ or pretending being a good firm and repaying the loan $\theta_b R(k_g) - (1+i_g)k_g$). The same should be true for the good type firm.

Finally, another constraint that any contract should satisfy is that expected (average) profits of financial intermediaries should not be negative. In order to define the expected profit of financial intermediary, we will use the indicator function $\chi_j(i_j, k_j; \lambda)$ to denote the range of interest rate and capital at which the firm of type j will not default (see non-default constraint 4):

$$\chi_j(i_j, k_j; \lambda) = \begin{cases} 1 & \text{if } \theta_j R(k_j) - (1+i_j)k_j \geq (1-\lambda)\theta_j R(k_j) \\ 0 & \text{if } \theta_j R(k_j) - (1+i_j)k_j < (1-\lambda)\theta_j R(k_j) \end{cases} \iff$$

$$\chi_j(i_j, k_j; \lambda) = \begin{cases} 1 & \text{if } \lambda\theta_j R(k_j) \geq (1+i_j)k_j \\ 0 & \text{if } \lambda\theta_j R(k_j) < (1+i_j)k_j \end{cases}$$

The above indicator function means that if $\chi_j = 1$, the firm fulfills the contract, while if $\chi_j = 0$, the firm defaults. Thus, the expected profit by financial intermediaries is as follows:

$$\begin{aligned} &\nu \left[\chi_g(1+i_g)k_g + (1\text{-}\chi_g)(\lambda\text{-}\psi)\theta_g R(k_g)\text{-}(1+r)k_g\right] + \\ &(1\text{-}\nu)\left[\chi_b(1+i_b)k_b + (1\text{-}\chi_b)\left[(\lambda\text{-}\psi)\theta_b R(k_b)\right]\text{-}(1+r)k_b\right] \geq 0 \end{aligned} \tag{6}$$

The revenue of a financial intermediary from a contract is equal to $(1+i_j)k_j$ when the firm fulfills the contract ($\chi_j = 1$) and $\lambda\theta_j R(k_j)$ when the firm does not fulfill the contract ($\chi_j = 0$). The costs that a contract generate for the financial intermediary consist in the payment to depositors, $(1+r)k_j$, plus the bankruptcy costs when the firm defaults ($\chi_j = 0$), which are $\psi\theta_j R(k_j)$. Note that the bankruptcy cost is always incurred by the financial intermediary. This fact is not incompatible, as we will see later, with the outcome that the financial intermediary shifts the bankruptcy cost to firms through higher interest rates.

Definition 2 *A menu of contracts* $\{(i_g, k_g), (i_b, k_b)\}$ *is an equilibrium menu of contracts if it satisfies the following conditions:*

1. The financial intermediary non-negative profit condition (6) holds.

2. The revelation constraints (5) holds.

3. $\forall j \in \{g, b\}$ there is no other contract (i'_j, k'_j) for the type j in which the firm j is better off $\theta_j R(k'_j) - \min\left\{\lambda\theta_j R\left(k'_j\right),\ (1+i'_j)k'_j\right\} > \theta_j R(k_j) - \min\left\{\lambda\theta_j R(k_j), (1+i_j)k_j\right\}$ and in which financial intermediary gets at least zero profit $\chi_j(1+i_j)k_j + (1\text{-}\chi_j)\left[(\lambda\text{-}\psi)\theta_g R(k_j)\right] \geq (1+r)k_j$ and in which the revelation constraint (5) holds for $j' \neq j$.

4. There is no other menu of contracts $\{(i'_g, k'_g), (i'_b, k'_b)\}$, in which financial intermediary gets at least zero profit (6) and in which $\theta_j R(k'_j) - \min\left\{\lambda\theta_j R\left(k'_j\right), (i'_j+1)k'_j\right\} \geq \theta_j R(k_j) - \min\left\{\lambda\theta_j R(k_j), (1+i_j)k_j\right\}$ $\forall j$, being the last inequality strict for one of the types, and in which the revelation constraint (5) holds.

Thus, a menu of contracts is an equilibrium if it satisfies the non-negative financial intermediary profit condition (6) and the revelation constraint (5), and it is not possible to find, for one or both firm types, a better contract where non-negative financial intermediary profit condition (6) is satisfied. Note that the revelation constraint always holds when financial intermediary offers the same contract to both types. When the financial intermediary offers different contracts to different types, the revelation constraint should hold in order that the types self-select themselves and chose the contract that is designed for each of them.

When there is adverse selection, there are two possible types of contracts: i) the separating contract, which is commonly defined in the literature as an equilibrium in which each type has a different contract and; ii) the pooling contract, in which all the types have the same contract and in which one of the type, typically the more productive type, is cross-subsidizing the other type. A standard result in the literature of adverse selection is that the pooling contract is never equilibrium. The intuition of this result is that, the more productive type is cross-subsidizing the less productive. Thus, it is always possible that a principal offers a contract in which the more productive type is better off than in the pooling contract and the less productive type is worse off. Note that the reason why separating equilibrium may exist and pooling equilibrium does not is not because the separating equilibrium offers a different contract to each type and the pooling contract offers only one type of contract. The crucial reason is that in the separating equilibrium there is no cross-subsidy across types and in the pooling equilibrium there is cross-subsidy across types. This is the reason why we introduce in the paper a more general definition of pooling and separating equilibrium than in other papers of the literature. More precisely, we define the separating equilibrium as an equilibrium in which the zero profit condition of financial intermediary holds separately for each type of firm and the pooling equilibrium as a contract in which both firm types receive the same contract and the zero profit conditions do not hold separately for each firm type. That is, the zero profit condition holds jointly for all the types but the expected profit of financial intermediary for one of the types is negative. This means that the pooling equilibrium, according with our definition, is an equilibrium menu of contracts in which one of the types cross-subsidizes to the other type. The particular framework of this paper implies that it is possible to have a separating equilibrium in which the financial intermediary (the principal) offers the same contract to both firms types (the agents), as we will see in section 4. Given our definition of separating equilibrium, this does not contradict the standard result which says that there is no equilibrium in which one agent cross-subsidizes others, what we call pooling equilibrium.

Definition 3 *A menu of contracts $\{(i_g, k_g), (i_b, k_b)\}$ is a separating equilibrium if it is an equilibrium as defined in definition 2 and the financial intermediary non-negative profit condition (6) holds separately for each type. That is, the following equation holds:* $\forall j \in \{g, b\} \quad \chi_j(1+i_j)k_j + (1-\chi_j)(\lambda-\psi)\theta_j R(k_j)-(1+r)k_j \geq 0.$

Definition 4 *A menu of contracts $\{(i_g, k_g), (i_b, k_b)\}$ is a pooling equilibrium if it is an equilibrium as defined in definition 2 in which both agent receive the same contract $(i_g, k_g) = (i_b, k_b)$ and in which the financial intermediary non-negative profit condition (6) does not* *hold separately for each type. That is,* $\exists j \in \{g,b\} \quad \chi_j(1+i_j)k_j + (1\text{-}\chi_j)(\lambda\text{-}\psi)\theta_j R(k_j)\text{-}(1+r)k_j < 0$.

Lemma 5 *At equilibrium the zero profit condition of financial intermediary holds separately for each type of firm:* $\forall j \in \{g,b\}$ $\chi_j(1+i_j)k_j + (1\text{-}\chi_j)(\lambda\text{-}\psi)\theta_j R(k_j) = (1+r)k_j$. *(proof in appendix)*

The intuition of the above lemma is that if a financial intermediary offers a contract to a type making profits from it, then it is always possible that other financial intermediary can offer a contract to that type in which the firm is better off. Thus, it is not possible either that the financial intermediary has positive profits or that one type cross-subsidizes the other type. Thus, there is no pooling equilibrium.

Corollary 6 *The equilibrium is never a Pooling equilibrium.*

Lemma 7 *There is no default at equilibrium.(proof in appendix)*

Thus, default can never be better than fulfilling the contract. The intuition is that since financial intermediary makes zero profits and the equilibrium is separating, the bankruptcy cost in case of default is passed to the defaulting firm at the equilibrium. Thus, if there is a contract in which the firm defaults, it is always possible to offer a contract which the firm will fulfill, sparing the bankruptcy cost, which makes the firm better off.

Note that the fact that there is not default at equilibrium does not mean that the non default constraint is irrelevant. The reason is that the non default constraint implies a borrowing limit: in order to guarantee that firms do not default, financial intermediaries only lend to the firms up to the limit in which firms do not have incentives to default, called the non default borrowing limit, as we will see below.

Lemmas 5 and 7 imply the following corollary

Corollary 8 *The interest rates of contract for both type of firms, good and bad, are equal to the deposit interest rate* $i_b = i_g = r$.

Since financial intermediaries have zero profits in the contract of each type (lemma 5), and default does not occur at equilibrium (lemma 7), it follows that the interest rate that financial intermediary charges should be equal to the depositors interest rate. The above corollary implies also that the non-default constraint may be rewritten as follows:

$$\theta_j \, R\,(k_j) - (1+r)k_j \,\geq\, (1-\lambda)\theta_j \, R\,(k_j) \quad \forall j \,\in\, \{g,b\} \tag{7}$$

Figure 1 displays the non-default constraint (7): the firm has incentive to fulfill the contract when the curve that represents the profits when fulfilling the contract, $\theta_j R\,(k_j) - (1+r)k_j$, is above the curve that represents the profits when defaulting $(1-\lambda)\theta_j R\,(k_j)$. The non-default constraint may be rewritten as follows:

$$\lambda\theta_j \frac{R\,(k_j)}{k_j} \geq (1+r) \quad \forall j \,\in\, \{g,b\} \tag{8}$$

The above constraint means that the firm will fulfill the contract if the average payment (return per unit of lent capital) to financial intermediaries when the firm defaults is higher than when the firm fulfills the contract. This constraint is also displayed in figure 1. It follows from the properties of the revenue function $R(k)$ that the average revenue $R(k)/k$ is a decreasing function in k[9]. It is apparent from figure 1 that the non-default constraint implies a borrowing limit, defined as the level of capital at which the non-default condition is satisfied with equality and denoted by k_j^{nd}:

$$k_j^{nd} \overset{Def}{\Leftrightarrow} \lambda\theta_j \frac{R\left(k_j^{nd}\right)}{k_j^{nd}} = (1+r)$$

Since the average revenue is decreasing, the default constraint is satisfied when the amount of capital that the firm j receives is lower than k_j^{nd} (as shown in figure 1):

$$\lambda\theta_j \frac{R(k_j)}{k_j} \geq (1+r) \Leftrightarrow k_j \leq k_j^{nd} \quad \forall j \in \{g, b\} \tag{9}$$

Thus, k_j^{nd} is the maximum borrowing limit such that the firm does not default. We will call k_j^{nd} the non-default borrowing limit. Abusing of notation, let's define $k_j^{nd}(r, \lambda)$ as the function that relates the non-default borrowing limit k_j^{nd} of firm type j with the interest rate r and the degree of law enforceability λ:

$$k_j^{nd}(r, \lambda) \overset{Def}{\Leftrightarrow} \lambda\theta_j \frac{R\left(k_j^{nd}(r, \lambda)\right)}{k_j^{nd}(r, \lambda)} = (1+r) \tag{10}$$

Since the average revenue of capital $R(k)/k$ is a continuous, differentiable and decreasing function in $\Re_{++}$, the Implicit Function Theorem entails that the function $k^{nd}(.)$ is well defined, continuous and differentiable in $\Re_{++}$. Furthermore, the non-default borrowing limit increases with the degree of law enforceability λ and decreases with the interest rate r. Thus, as displayed in figure 2, the non-default borrowing limit depends on three factors:

- The degree of law enforceability λ: a rise in the degree of law enforceability increases the payment to financial intermediary in case of default (see left hand side

[9]Note that:

$$\frac{\partial \left[\frac{R(k)}{k}\right]}{\partial k} = \left[R'(k)k - R(k)\right]\frac{1}{k^2}$$

Using the Taylor Theorem and the fact that $R(k)$ is concave and $R(0) = 0$:

$$\begin{aligned} R(0) &= R(k) - R'(k)k + \frac{1}{2}R"(\alpha k)k^2 \Leftrightarrow \\ R'(k)k - R(k) &= -R(0) + \frac{1}{2}R"(\alpha k)k^2 = \frac{1}{2}R"(\alpha k)k^2 < 0 \end{aligned}$$

where α is a constant between 0 and 1.

of equation 8 and figure 2.a below), which implies lower profits for the firm in case of default (see right hand side of equation 7 and figure 2.a above). Thus, better law enforcement encourages the fulfillment of the contract, increasing the non-default borrowing limit.

- Productivity θ: higher productivity implies larger payment to financial intermediary in case of default (see left hand side of equation 8 and figure 2.b below). Thus, higher productivity encourages the fulfillment of the contract, increasing the non-default borrowing limit.

- The interest rate r: a rise in the interest rate increases the payment to financial intermediary in case of fulfilling the contract (see right hand side of equation 8 and figure 2.c below), which implies lower profits for the firm in case of fulfilling the contract (see left hand side of equation 7 and figure 2.a above). Thus, a higher interest rate deters the fulfillment of the contract, reducing the non-default borrowing limit.

Since higher productivity encourages the fulfillment of the contract, the non-default borrowing limit is higher for the good type than for the bad type:

$$k_g^{nd}(r,\lambda) > k_b^{nd}(r,\lambda)$$

Thus, as the good type firm has higher productivity, she gets a larger amount of capital than the bad type firm at the same interest rate. Thus, if financial intermediaries would have information about the firm types, the good type firm would get a better contract than the bad type: it receives more capital and pays the same interest rate as the bad type firm. This means that the good type firm has never the incentive to pretend to be a bad type. Only the bad type firm has incentives to not reveal her type since the non default borrowing constraint is looser for good type firm. Thus, the capital that bad firms choose is either the optimal capital (if it is possible), or the non-default borrowing limit:

$$k_b(r,\lambda) = \min\left\{k_b^{nd}(r,\lambda), k_b^*(r)\right\}$$

Hence, the profit function of bad firms is as follows:

$$\pi_b(r,\lambda) = \max_{k\in\left[0,k_b^{nd}(r,\lambda)\right]} \theta_b R(k) - (1+r)k = \theta_b R(k_b(r,\lambda)) - (1+r)k_b(r,\lambda) \tag{11}$$

With this, we characterize the optimal contract for the bad type.

For the good type, we need to take into account both the non-default constraint and the revelation constraint; we will start with the latter. We have seen that, due to perfect competition in the financial sector, financial intermediaries offer to the bad type firm the best possible contract such that the financial intermediary gets zero profit and the non-default constraint is satisfied for the bad type firm. This means that if a firm of the bad type asks for the contract of the good type, it is because she plans to default. If she wouldn't plan to default, she would be better off accepting the contract offered to her own type.

Lemma 9 *If the revelation constraint of the bad type is binding, then* $\max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\} = (1-\lambda)\theta_b R(k_g)$. *That is, the bad type can only prefer the contract for the good type if she defaults. (proof in appendix)*

Good and bad firms always pay the same interest rate, however good firms receive more capital than bad firms when law enforceability is effective enough. This does not mean that, when bad firms receive lower capital, it is necessarily because they are identified as bad type (and refused to be given more capital). In fact, when law enforceability is effective enough, bad firms chose at equilibrium a lower level of capital than good firms and revelation constraint is not binding. The problem arises when the law enforcement becomes weak. In this case bad firms would get higher profits getting the same amount of capital as good type firms and defaulting, rather than getting its optimal level of capital and fulfilling the contract. When this happens, the revelation constraint is binding and affects the allocation of capital at equilibrium. Thus, the revelation constraint for the good type is never binding and the revelation constraint for the bad type is as follows:

$$\theta_b R(k_b) - (1+r)k_b \geq (1-\lambda)\theta_b R(k_g) \tag{12}$$

Let's define the revelation borrowing limit k^{rv} as the capital offered to the good type at which the revelation constraint is satisfied with equality for a given k_b:

$$k^{rv} \overset{def}{\Leftrightarrow} (1-\lambda)\theta_b R(k^{rv}) = \pi_b = \theta_b R(k_b) - (1+r)k_b \Leftrightarrow R(k^{rv}) = \frac{\pi_b}{(1-\lambda)\theta_b}$$

Since the revenue function $R(k)$ is strictly increasing, the revelation borrowing limit is well defined. Furthermore, it follows from the Implicit Function Theorem and (11) that the revelation borrowing limit is an increasing function of the degree of law enforceability λ and a decreasing function of the interest rate r. We will denote such function $k^{rv}(r,\lambda)$:

$$k^{rv}(r,\lambda) \overset{def}{\Leftrightarrow} (1-\lambda)\theta_b R\left(k^{rv}(r,\lambda)\right) = \pi_b(r,\lambda) \tag{13}$$

It follows from the fact that the revenue function $R(k)$ is increasing that the revelation constraint is satisfied only if the amount of capital is smaller than the revelation borrowing limit:

$$\theta_b R(k_b) - (1+r)k_b \geq (1-\lambda)\theta_b R(k) \Leftrightarrow k \leq k^{rv}$$

Figures 3, 4 and 5 display the revelation constraint. The revelation borrowing limit depends on:

- The degree of law enforceability λ:. There are two effects of the degree of law enforceability: i) A direct effect: an increase in the degree of law enforceability reduces the profits of bad firms when not revealing their true type, and hence diminishes the incentives to dissemble their type. Thus, better law enforcement increases revelation borrowing limits. ii) Furthermore, there is an indirect effect: if bad firms are constrained by the non-default borrowing limit, a rise in λ increases the non-default borrowing limit and the profits of bad firms when they truly reveal their type, thus increasing the revelation borrowing limit.

- The interest rate r: an increase in the interest rate reduces the profits of bad firms, and therefore the payment of bad firms when they truly reveal their type, reducing their incentive to do so.

Lemma 10 *If the non-default constraint is binding for the good type, then the revelation constraint of the bad type is not satisfied. (proof in appendix)*

The above lemma means that the revelation constraint is always tighter for the good firm than the non-default constraint. In other words, the revelation constraint is the one which is relevant for the good firms.

Corollary 11 *The non-default constraint is never binding for good firms.*

Thus, when the non-default constraint is binding for good firms, the revelation borrowing limit is smaller than the non-default limit. Since the revelation constraint is always more restrictive that the non-default constraint, the non-default constraint is never binding for the good type in equilibrium.

To sum up, if $\{(i_g, k_g), (i_b, k_b)\}$ is a menu of contracts, then $i_g = i_b = r$ and k_b and k_g should be the solutions of the following optimization problems:

$$\begin{aligned} k_b = \quad & \arg\max_k \theta_b R(k) - (1+r)k \\ & s.t.\ \theta_b R(k) - (1+r)k \geq (1-\lambda)\theta_b R(k) \end{aligned} \tag{14}$$

$$\begin{aligned} k_g = \quad & \arg\max_k \theta_g R(k) - (1+r)k \\ & s.t.\ \theta_b R(k_b) - (1+r)k_b \geq (1-\lambda)\theta_b R(k) \end{aligned} \tag{15}$$

An equivalent way to rewrite the above menu of contracts is as follows:

$$\begin{aligned} k_b = \quad & \arg\max_k \theta_b R(k) - (1+r)k \\ & s.t.\ k_b \leq k_b^{nd} \end{aligned} \tag{16}$$

$$\begin{aligned} k_g = \quad & \arg\max_k \theta_g R(k) - (1+r)k \\ & s.t.\ k_g \leq k^{rv} \end{aligned} \tag{17}$$

With these, we fully characterise the optimal menu of contracts.

4. Law Enforcement and Firm Size Distribution

In this section we examine how the degreee of law enforcement affects the equilibrium menu of contracts found before. We have seen in the previous section that better law enforceability loosens the incentive constraints since it reduces the payment of firms when they default or do not truly reveal their type. This means that better law enforceability expands the borrowing limits of firms, both the non-default borrowing limit (which affects bad firms) and the revelation borrowing limit (which affects good firms). In this section we analyze the relationship between the degree of law enforceability and the firm size distribution.

Let's define λ_j^{nd} as the degree of law enforceability at which the type j firm satisfies the non-default constraint (7) with equality for the optimal level of capital (defined in 1):

$$\lambda_j^{nd} \overset{Def}{\Leftrightarrow} \theta_j R\left(k_j^*\right) - (1+r)k_j^* = (1-\lambda_j^{nd})\theta_j R\left(k_j^*\right) \Leftrightarrow \lambda_j^{nd} \equiv \frac{(1+r)k_j^*}{\theta_j R\left(k_j^*\right)}$$

We will call λ_j^{nd} the non-default degree of law enforceability for the type j. If the degree of law enforceability λ is above the non-default degree of law enforceability for the type j, then this type of firm does not have incentives to default when she maximizes profits (she chooses the optimal capital k_j^*). When λ is below λ_j^{nd}, firm of type j will default if she is allowed to chose any amount of capital without constraints. This is why, if λ is below λ_j^{nd}, firm of type j will be constrained at equilibrium. Using the definition of optimal capital (see equation 1), it yields:

$$\lambda_j^{nd} = \frac{R'\left(k_j^*\right)k_j^*}{R\left(k_j^*\right)} = \varepsilon^R\left(k_j^*\right) < 1 \tag{18}$$

where $\varepsilon^R\left(k_j\right)$ denotes the elasticity of firm revenue with respect to capital, and the inequality comes from the assumption that $R(k)$ is a strictly concave function and $R(0) = 0$.

Let's define λ^{rv} as the the degree of law enforceability at which the revelation constraint (12) is satisfied with equality for the optimal level of capital (defined in equation 1):

$$\begin{aligned} \lambda^{rv} &\overset{Def}{\Leftrightarrow} \theta_b R(k_b^*) - (1+r)k_b^* = (1-\lambda^{rv})\theta_b R(k_g^*) \Leftrightarrow \\ \lambda^{rv} &= \frac{\theta_b R(k_g^*) - [\theta_b R(k_b^*) - (1+r)k_b^*]}{\theta_b R(k_g^*)} < 1 \end{aligned}$$

We will call λ^{rv} the revelation degree of law enforceability. Using the definition of optimal capital (1) and the definition of non-default degree of law enforceability (18), it follows that the revelation degree of law enforceability λ^{rv} is larger than the non-default degree of law enforceability for the bad type:

$$\begin{aligned} \lambda^{rv} &= \frac{R(k_g^*) - R(k_b^*)}{R(k_g^*)} + \frac{(1+r)k_b^*}{\theta_b R(k_g^*)} = \frac{R(k_g^*) - R(k_b^*)}{R(k_g^*)} + \frac{\theta_b R'(k_b^*)k_b^*}{\theta_b R(k_g^*)} \\ \lambda^{rv} &= \frac{R(k_g^*) - R(k_b^*)}{R(k_g^*)} + \lambda_b^{nd}\frac{R(k_b^*)}{R(k_g^*)} = \frac{R(k_g^*) - R(k_b^*)}{R(k_g^*)}(1-\lambda_b^{nd}) + \lambda_b^{nd} > \lambda_b^{nd} \end{aligned} \tag{19}$$

where in the second equality we use the definition of optimal capital (see equation 1) and in the third equality we use the definition of non-default degree of law enforceability (equation 18). When λ is above λ^{rv}, bad firms do not have any incentive to accept the contract designed for the good type and, consequently, the revelation constraint is not binding at equilibrium. When λ is below λ^{rv}, revelation constraint is binding. Since λ^{rv} is larger than λ_b^{nd}, when the degree of law enforceability is in the interval $[\lambda^{rv}, 1]$ neither the non-default nor the revelation constraint are binding for the optimal capital of both firm types, which means that the efficient allocation is the equilibrium. Figure 3 displays the case in which the degree of law enforceability is in the interval $[\lambda^{rv}, 1]$. In such case neither the non-default limit for the bad firms, nor the revelation borrowing limit for good firms, are binding. Figure 3 displays also the profits of bad firms when they decide not to truly reveal their type and the degree of law enforceability is equal to λ^{rv} (see dotted line). In this last case the revelation borrowing limit coincides with the optimal capital for the good type. We may summarize these results in the following proposition:

Proposition 12 *When $\lambda \in [\lambda^{rv}, 1]$, neither the revelation nor the non-default constraints are binding, thus $k_b = k_b^*$ and $k_g = k_g^*$. That is, the firm size distribution is efficient.*

When the degree of law enforceability is in the interval $\left[\lambda_b^{nd}, \lambda^{rv}\right)$, the non-default constraints are not binding for the optimal capital of both types, but the revelation constraint is binding. This implies that the the size of the bad type firm is optimal: it receives at equilibrium its optimal amount of capital. The good type receives an inefficiently small amount of capital (smaller than its optimal level) that is determined by the revelation constraint (that is, by the revelation borrowing limit: $k_g = k^{rv}$). Thus the size of the good firms is inefficiently small. Figure 4 displays the case in which the degree of law enforceability is in the interval $\left[\lambda_b^{nd}, \lambda^{rv}\right)$. In this case the non-default limit is not binding for the bad firms, and therefore the capital of these firms reaches its optimal level. However, good firms are constraint by the revelation borrowing limit, which is tighter when law enforcement is weaker (λ is smaller).

Since the revelation borrowing limit k^{rv} is increasing in the degree of law enforceability (see 13), the amount of capital that good firms receive is increasing in the degree of law enforceability. We may summarize these results in the following proposition:

Proposition 13 *When $\lambda \in \left[\lambda_b^{nd}, \lambda^{rv}\right)$ the non-default constraint is not binding but the revelation constraint is binding, thus $k_b = k_b^*$ and $k_g = k^{rv} < k_g^*$. That is, the size of bad firms is optimal while the size of good firms is inefficiently small. Furthermore, k^{rv} is increasing in λ.*

Note that according with the above proposition when $\lambda \in \left[\lambda_b^{nd}, \lambda^{rv}\right)$ the capital of the bad firm is the same as in the efficient allocation but the amount of capital of good firms is smaller. This implies that the fraction of capital devoted to good firms is smaller than in the efficient allocation:

$$\left.\begin{matrix} k_g < k_g^* \\ k_b = k_b^* \end{matrix}\right\} \Rightarrow \frac{k_b}{k_g} > \frac{k_b^*}{k_g^*} \Rightarrow$$

$$\frac{\nu k_g}{\nu k_g + (1-\nu)k_b} = \frac{\nu}{\nu + (1-\nu)\frac{k_b}{k_g}} < \frac{\nu}{\nu + (1-\nu)\frac{k_b^*}{k_g^*}} = \frac{\nu k_g^*}{\nu k_g^* + (1-\nu)k_b^*}$$

Thus, it follows from proposition 13 that good firms receive an inefficiently small portion of capital:

Corollary 14 *When $\lambda \in \left[\lambda_b^{nd}, \lambda^{rv}\right)$ good firms receive an inefficiently small share of the capital: $\frac{\nu k_g}{\nu k_g + (1-\nu)k_b} < \frac{\nu k_g^*}{\nu k_g^* + (1-\nu)k_b^*}$.*

When $\lambda < \lambda_b^{nd}$, the optimal capital of bad firms does not satisfy anymore the non-default constraint. The optimal contract of the bad firm (14) may be rewritten as follows:

$$\max_{k \in \left[0, k_b^{nd}\right]} \theta_b R(k) - (1+r)k$$

Since the revenue function is concave and the non-default borrowing limit is smaller than the optimal capital, the capital of the bad type is her non-default borrowing limit:

$$k_b = k_b^{nd}$$

Since we have seen that $\lambda^{rv} > \lambda_b^{nd}$, when $\lambda < \lambda_b^{nd}$ the revelation constraint is binding for the good type:

$k_g = k^{rv}$

Lemma 15 *If $k_b = k_b^{nd} \leq k_b^*$ then $k_g = k^{rv} = k_b^{nd}$.*

The above lemma says that when the non-default constraint is binding for the bad type, the revelation borrowing limit is equal to the non-default borrowing limit of the bad type. That is, when the non-default borrowing constraint is binding, the borrowing limit of both the good and the bad type is the non-default borrowing limit of the bad type. Thus, when $\lambda < \lambda_b^{nd}$ the capital of the good and bad firms is the same, and equal to the non-default borrowing limit of the bad type. Such limit is increasing in the degree of law enforceability λ. Figure 5 displays the case in which the degree of law enforceability is smaller than λ_b^{nd}. In this case the non-default constraint is binding for the bad firms and therefore, the capital of these firms is equal to the non-default borrowing limit. The revelation constraint is binding for the good firms and their capital is equal to the revelation borrowing limit. It turns out that the non-default borrowing limit for bad firms coincides with the revelation borrowing limit. Figure 5 shows that when bad firms are borrowing-constrained, their profits when concealing their type are superior to their profits when revealing their type, for any capital larger than the non-default borrowing limit. Thus, the revelation borrowing limit coincides with the non-default borrowing limit for the bad type. Figure 5 displays also the profits of bad firms when they decide not to reveal their type and the degree of law enforceability is equal to λ_b^{nd} (see dotted line). In this last case the revelation borrowing limit and the non-default borrowing limit for the bad type coincide with the optimal capital for the bad type. These results may be summarized in the following proposition.

Proposition 16 *When $\lambda \in \left(0, \lambda_b^{nd}\right)$ the non-default constraint of the bad type and the revelation constraint are binding. Furthermore, $k_b = k_g = k_b^{nd} < k_b^*$. That is, the firm size of both good and bad type is inefficiently small. Finally, k_b^{nd} is increasing in λ.*

Note that according with the above proposition when $\lambda \in \left(0, \lambda_b^{nd}\right)$ the capital of the bad and good firms are the same, but the capital of the good type firm is larger than the capital of the bad type firm in the efficient allocation. This implies that the fraction of capital devoted to good firms is smaller than in the efficient allocation:

$$\left.\begin{array}{l} k_b^* < k_g^* \\ k_b = k_g \end{array}\right\} \Rightarrow \frac{k_b^*}{k_g^*} < 1 = \frac{k_b}{k_g} > \frac{k_b^*}{k_g^*} \Rightarrow$$

$$\frac{\nu k_g}{\nu k_g + (1-\nu) k_b} = \frac{\nu}{\nu + (1-\nu)\frac{k_b}{k_g}} < \frac{\nu}{\nu + (1-\nu)\frac{k_b^*}{k_g^*}} = \frac{\nu k_g^*}{\nu k_g^* + (1-\nu) k_b^*}$$

Thus, it follows from proposition 16 that good firms receive an inefficiently small portion of capital:

Corollary 17 *When $\lambda < \lambda_b^{nd}$ good firms receive an inefficiently small share of the capital: $\frac{\nu k_g}{\nu k_g + (1-\nu)k_b} < \frac{\nu k_g^*}{\nu k_g^* + (1-\nu)k_b^*}$.*

Summarizing, the allocation of capital to firms depends on the degree of law enforceability λ. There are three different types of equilibrium:

1. When law enforcement is effective enough, $\lambda > \lambda^{rv}$, incentive constraints are not binding and both good and bad firms chose their optimal level of capital. That is, firm size distribution is optimal.

2. When law enforcement is in a middle interval, $\lambda \in \left[\lambda_b^{nd}, \lambda^{rv}\right)$, bad firms are not constrained but good firms are constrained by the revelation borrowing limit. This means that bad firms choose their optimal capital but good firms are underfinanced, that is, their firm size is smaller than their optimal one. An improvement in law enforcement, loosens the revelation constraint and increases the capital allocated to good firms.

3. When law enforcement is weak, $\lambda < \lambda_b^{nd}$, bad firms are constrained by the non-default borrowing limit and good firms are constrained by the revelation borrowing limit. This means that both bad and good firms are underfinanced and are smaller than their optimal size. Furthermore, good firms receive an inefficiently small share of the capital. An improvement in law enforcement loosens the default and revelation constraints and increases the capital allocated to both firms.

5. Existence of Equilibrium

Corollary 6 says that pooling equilibrium does not exist, where pooling equilibrium is defined as a menu of contracts in which both firm types receive the same contract and there is cross-subsidy across agents. As we have explained, the intuition of this result is that when good agents are cross-subsidizing bad agents, it is always possible by the principal to offer to the good agent a better contract than the pooling contract. Furthermore, separating equilibrium may not exist either. The reason is that, under certain circumstances, it is possible to offer a pooling contract in which agents are better off than in the separating contract. When this happens, it does not exist any equilibrium contract. In order to rule out this undesired possibility, in this section we analyze the pooling contract and we compare the profits of the good firm in the best of such contracts with the profits of the good firm at the (separating) equilibrium, to determine whether the (separating) equilibrium exists. The conclusion is that if the bankruptcy cost ψ is high enough, the equilibrium always exists.

To check whether (separating) equilibrium exists we will compare the separating equilibrium with the best possible pooling contract. Such pooling contract will be called pooling quasi-equilibrium and it is defined as follows.

Definition 18 *A contract (i,k) is a pooling quasi-equilibrium contract if it satisfies the following conditions:*

1. The financial intermediary non-negative profit condition (6) holds;

2. The financial intermediary non-negative profit condition (6) does not hold separately for each type. That is, $\exists j \in \{g,b\} \quad \chi_j(1{+}i_j)k_j + (1{-}\chi_j)(\lambda{-}\psi)\theta_j R(k_j){-}(1{+}r)k_j < 0$;

3. There is no other pooling contract (i'_j, k'_j) in which the financial intermediary non-negative profit condition (6) holds, but not separately, $\exists j \in \{g, b\}$ $\chi_j(1+i'_j)k'_j+(1-\chi_j)(\lambda-\psi)\theta_j R(k'_j)-(1+r)k'_j < 0$ and in which both agents are better off, $\forall j \in \{g, b\}$ $\theta_j R(k') - (1+i')k' \geq \theta_j R(k) - (1+i)k$, being the last inequality strict for one of the types.

Thus, a pooling contract is a pooling quasi-equilibrium if there is no another pooling equilibrium in which the agents are better off, being one of them strictly better off.

Lemma 19 *In any pooling quasi-equilibrium (k, i) the bad type firm defaults: $\theta_b R(k) - (1+i)k < (1-\lambda)\theta_b R(k)$. (proof in appendix)*

We have defined the pooling contract as a menu of contracts in which both types have the same contract and there is cross-subsidizing across agents (see definition 4). Note that if there is no bankruptcy in equilibrium, the zero profit condition of financial intermediaries implies that the interest rate is equal to the deposit interest rate (see corollary 8), which implies that there is no cross-subsidy across agents, which means that the contract is not a pooling equilibrium according to our definition. Thus, the only way in our model to have cross-subsidy across agents is that one of the types defaults and the financial intermediary passes the unpaid debt of the defaulter plus the bankruptcy cost to the other type. Since the bad type has less revenues and consequently pays less in case of default, such type always has more incentives to default than the good type.

Lemma 20 *In any pooling quasi-equilibrium (k, i) the good type firms fulfill the contract: $\theta_b R(k) - (1+i)k \geq (1-\lambda)\theta_b R(k)$. (proof in appendix)*

The intuition of the above lemma is that given the fact that the bad type is going always to default (see lemma 19) and given that her revenues are smaller than those of the good type, the bad type is going to pay always less than the good type. This means that the good type is going to cross-subsidize the bad type. Thus, the financial intermediary is going to pass the bankruptcy cost of the bad type to him. If the good type defaults as well, the financial intermediary will pass to the good type both the bankruptcy cost of the bad type and her own bankruptcy cost. This means that if there is a pooling contract in which the good type defaults, it is always possible to find another contract in which the good type does not default and she is better off due to the fact that she spares herself her own bankruptcy costs.

Note that lemmas 19 and 20 together imply that the good type cross-subsidizes the bad type and that the financial intermediary passes the bankruptcy costs of the bad type to the good type through higher interest rates.

The separating contract is not an equilibrium if there is a pooling contract in which the good type is strictly better off than in the separating contract. Thus, to prove existence of (separating) equilibrium, it is enough to prove that the profit of the good type firm in the separating contract is higher than in the best possible pooling quasi-equilibrium. We have seen in lemmas 19 and 20 that in any pooling quasi-equilibrium the good type fulfills the contract and the bad type defaults. Thus, the best pooling quasi-equilibrium for the good type is as follows[10]:

$$\pi_g^{Pooling}(\lambda,\psi) = \begin{array}{rl} \max\limits_{k,i} \theta_g R(k) - (1+i)k & \\ s.t.\ \theta_g R(k) - (1+i)k \geq (1-\lambda)\theta_g R(k) & \\ (1+r)k \leq (1-\nu)(\lambda-\psi)\theta_b R(k) + \nu(1+i)k & \end{array} \tag{20}$$

The first of the above constraints is the non-default constraint for the good type, since we have proven in lemma 20 that good types do not default at pooling quasi-equilibria. The second constraint is the zero profit condition for financial intermediaries, which takes account of lemmas 19 and 20 that in any pooling quasi-equilibrium the good type fulfills the contract and the bad type defaults.

Proposition 21 *For any ν there is $\underline{\psi}$ such that if $\psi > \underline{\psi}$ then $\forall\lambda \in [0, \lambda^{rv}]$, $\pi_g^{Separating}(\lambda) \geq \pi_g^{pooling}(\lambda,\psi)$. (proof in appendix)*

Thus, when the bankruptcy costs are high enough, the separating contract is an equilibrium.

The results of this section are summarized in the following table:

6. General Equilibrium

In this section we extend the model in order to analyze the effect of imperfect law enforcement on firm size distribution in a general equilibrium environment. This approach will allow us to analyze the effect of law enforcement on per capita production and aggregate productivity. In order to do this, we keep all the assumptions in the model, stated in section 2, and we add two elements needed in order to extend the model from a partial to a general equilibrium framework: i) We incorporate to the model agents that own capital. We need this modification in order to determine the interest rate r which is no longer an exogenous variable, being determined in the capital market in which the demand of capital by firms should be equal to this supply of capital. We have analyzed the demand of capital by firms but we have not introduced yet the supply of capital. In order to do this we need to incorporate to the model agents that own capital. ii) We will introduce a sunk cost. The reason is that the firms technology up to now presents decreasing return to scale, which means that it does not have an optimal size in general equilibrium: the lower the amount of capital of each firm, the more productive each firm is. The introduction of a sunk cost will imply that each firm type has an optimal firm size and, consequently, there exists a well defined efficient firm size distribution.

[10] We could allow the financial intermediary to use mixed strategies, in which case the best pooling cuasi-equilibrium would be as follows:

$$\begin{array}{l} \max\limits_{k,i,\mu} \theta_g R(k) - (1+i)k \\ \quad s.t.\ \theta_g R(k) - (1+i)k \geq (1-\lambda\mu)\theta_g R(k) \\ \quad\quad (1+r)k \leq (1-\nu)\mu(\lambda-\psi)\theta_b R(k) + \nu(1+i)k \end{array}$$

where μ is the probability that the financial intermediary incurs bankruptcy costs and obliges defaulters to pay the fraction λ of their revenues. Since the results do not change at all with mixed strategies, we do not include them in the paper in order to simplify the exposition.

We will assume that there is a continuum of risk-neutral agents indexed in the interval $[0, 1]$, each of them with k units of capital. Thus, k is also the per capita capital. Agents may chose between depositing their funds with a financial intermediary or creating a firm. We will refer to the agents that chose to create a firm as entrepreneurs and to the ones that deposit their capital with a financial intermediary as depositors. To create a firm, a sunk cost of k units is required[11]. After investing the sunk cost in creating a firm, entrepreneurs receive a productivity shock $\theta_j \in \{\theta_g, \theta_b\}$. Agents decide whether to create a firm or not before knowing the productivity shock that the firm would receive. Given that agents are risk neutral, the expected profits of creating a firm should be equal in equilibrium to the expected depositors´ income. Otherwise, either all agents will decide to become entrepreneurs, if the expected profits of the firms are higher than expected depositors' income, or to become depositors, in the opposite case. Thus, the expected profits of a firm should be equal to the depositors' income at equilibrium:

$$\nu\, \pi_g + (1 - \nu)\, \pi_b = (1 + r)k \tag{21}$$

where π_g and π_b are the profits of the good and bad firm in the menu of contracts (14) and (15):

$$\pi_j = \theta_j R(k_j) - (1 + r)k_j \qquad j \in \{g, b\}$$

In this setup, the number of firms and the interest rate are endogenous variables. We will refer to the per capita number of firms as n. Given that entrepreneurs should invest their k units of capital in order to create a firm, the capital market clearing condition will be as follows:

$$n\, [\nu\, k_g + (1 - \nu)\, k_b] = (1 - n)k \tag{22}$$

That is, there are n firms in per capita terms and the average demand of capital by these firm is $[\nu\, k_g + (1 - \nu)\, k_b]$. Thus the per capita demand of capital by firms is equal to $n\, [\nu\, k_g + (1 - \nu)\, k_b]$. Since the per capita number of entrepreneurs coincides with the per capita number of firms, n, and agents that are not entrepreneurs are depositor, there are $(1 - n)$ per capita depositors, each of them with k units of funds. This means that the per capita supply of capital is equal to $(1 - n)k$.

Definition An equilibrium is an allocation of resources $\{k_g, k_b, n\}$ and an interest rate r such that:

- k_g, k_b satisfy the menu of contracts defined in (14) and (15);
- The arbitrage condition (21) is satisfied;
- The capital market clearing condition (22) is satisfied.

[11] The assumption that the sunk cost is equal to the per capita amount of capital implies that the optimal per capita number of firms is constant, that is, it does not depend on the per capita amount of capital. This feature of the model would be able to be replicated in a neoclassical model with capital and labor in which the sunk cost would consist in a fixed amount of labor, as the one in subsection **??**.

6.1. The Benchmark case: The Efficient allocation

Let's define per capita income as the payment that agents receive net of their initial capital:

$$y = n\left[\nu\, [\theta_g R(k_g)\text{-}(1{+}r)k_g] + (1\text{-}\nu)\, [\theta_b R(k_b)\text{-}(1{+}r)k_b]\right] + (1\text{-}n)(1{+}r)k\text{-}k$$

That is, there are n per capita entrepreneurs and their payments are equal to profits which on average are equal to $\nu\, [\theta_g R(k_g)\text{-}(1{+}r)k_g]{+}(1\text{-}\nu)\, [\theta_b R(k_b)\text{-}(1{+}r)k_b]$. There are $1-n$ per capita depositors, their payments are equal to their capital k plus the interest derived from it rk. The sum of the payments of entrepreneur plus depositors, minus the capital is the income that may be rewritten as follows:

$$y = n\left[\nu\,\theta_g R(k_g) + (1-\nu)\theta_b R(k_b)\right] - k \tag{23}$$

We define the efficient allocation $\left(n^{*E}, k_g^{*E}, k_b^{*E}\right)$ as the one that maximizes per capita income subject to the feasibility constraint (where superscript E stands for general equilibrium):

$$\max_{n,k_g,k_b} n\left[\nu\theta_g R(k_g) + (1-\nu)\theta_b R(k_b)\right] - k$$
$$s.t.\text{: } n\left[\nu\, (k_g + k) + (1-\nu)\, (k_b + k)\right] \le k$$

The feasibility constraint means that the capital used by firms in per capita terms, which is equal to the per capita number of firms n multiplied by the average amount of capital used by firms, including both the capital used in production k_j and the sunk cost k, should be smaller than or equal as the per capita amount of capital k. The first order conditions of the above problem are:

$$\theta_g R'(k_g^{*E}) = \ell \tag{24}$$
$$\theta_b R'(k_b^{*E}) = \ell \tag{25}$$
$$\left[\nu\theta_g R(k_g^{*E}) + (1-\nu)\theta_b R(k_b^{*E})\right] = \ell\left[\nu\, k_b^{*E} + (1-\nu)\, k_b^{*E} + k\right] \tag{26}$$
$$n^*\left[\nu\, \left(k_g^{*E} + k\right) + (1-\nu)\, \left(k_b^{*E} + k\right)\right] = k \tag{27}$$

where ℓ is the Lagrangian multiplier. Equations (24), (25) and (26) imply that:

$$\theta_g R'(k_g^{*E}) = \theta_b R'(k_b^{*E}) \tag{28}$$
$$\nu\,\theta_g\left[R(k_g^{*E})\text{-}R'(k_g^{*E})k_g^{*E}\right] + (1\text{-}\nu)\,\theta_b\left[R(k_b^{*E})\text{-}R'(k_b^{*E})k_b^{*E}\right] = \theta_g R'(k_g^{*E})k \tag{29}$$
$$n^*\left[\nu\, \left(k_g^{*E} + k\right) + (1-\nu)\, \left(k_b^{*E} + k\right)\right] = k \tag{30}$$

Solving the above system of equations we would get the optimal firm size for bad and good type of firms, k_g^{*E} and k_b^{*E}, and the optimal per capita amount of firms n^*. It is easy to check that the efficient allocation coincides with the equilibrium allocation under perfect enforcement. In fact, if we interpret the Lagrangian multiplier as the price of the capital, $\ell = (1{+}r^*)$, we will obtain the equilibrium conditions under perfect law enforcement:

$$\theta_g R'(k_g^{*E}) = (1{+}r^*) \tag{31}$$
$$\theta_b R'(k_b^{*E}) = (1{+}r^*) \tag{32}$$
$$\nu\, \left[\theta_g R(k_g^{*E})\text{-}(1{+}r)k_g^{*E}\right] + (1\text{-}\nu)\, \left[\theta_b R({k_b}^{*E})\text{-}(1+r)k_b^{*E}\right] = (1{+}r^*)k \tag{33}$$
$$n^*\left[\nu\, k_g^{*E} + (1\text{-}\nu)\, k_b^{*E}\right] = (1\text{-}n^*)k \tag{34}$$

The first two equations, (31) and (32), say that firms maximize profits without borrowing constraints and consequently firms' marginal productivity of capital (or marginal revenue of capital) should be equal to the renting price of capital. The third equation is the non-arbitrage condition that says that the expected revenues of entrepreneurs should be equal to the capital income by depositors. The last equation is the capital market clearing condition. Equations (31), (32) and (33) imply the same system of equation that determine the efficient allocation, equations (28) to (30). Thus, when law enforcement is perfect, the equilibrium allocation coincides with the efficient one.

Remark 22 *Note that, in contrast to partial equilibrium, the optimal capital in general equilibrium, k_j^{*E}, does not depend on the interest rate, which is an endogenous variable in such context. We denote by $k_j^*(r)$ the function that relates the capital that maximizes the profits of the firm without constraints to the interest rate (defined in (1). We will call this $k_j^*(r)$ the* unconstrained demand for capital *by firm type j. Note that $k_j^*(r)$ is what we called optimal capital in partial equilibrium, when the interest rate was exogenous (analyzed in previous sections).*

6.2. Imperfect law enforcement

As we have seen in section 3, bad firms are constrained by the non-default borrowing limit, $k_b^{nd}(r,\lambda)$, defined in (10), which is a decreasing function of the interest rate r and an increasing function of the degree of law enforceability λ. $k_b^*(r)$ is the unconstrained demand for capital by bad type for a given interest rate, defined in (1), which is decreasing in the interest rate. Since bad firms only suffer the non-default constraint, we may define the demand function for capital by a bad firm as the minimum between the unconstrained demand for capital and the non-default borrowing limit:

$$k_b\left(r,\lambda\right) = \min\left\{k_b^{nd}\left(r,\lambda\right), {k_b}^*\left(r\right)\right\}$$

Thus, the demand for capital by a bad firm is decreasing in the interest rate r and increasing in the degree of law enforceability λ when the non-default borrowing limit is binding. The profits of bad firms are:

$$\pi_b(r,\lambda) = \max_{k\in\left[0,k_b^{nd}(r,\lambda)\right]} \theta_b R\left(k\right) - (1+r)k = \theta_b R\left(k_b(r,\lambda)\right) - (1+r)k_b(r,\lambda)$$

It follows from the envelope theorem that $\pi_b(r,\lambda)$ is a strictly decreasing function of r and does not depend on λ when the non-default constraint is not binding, and it is strictly increasing on λ when the non-default constraint is binding, $k_b^{nd}\left(r,\lambda\right) < {k_b}^*\left(r\right)$.[12]

As we have shown in section 3, good firms are constrained by the revelation borrowing limit, $k^{rv}(r,\lambda)$, defined in (13), which is a decreasing function of the interest rate r and an increasing function of the degree of law enforceability λ. Thus, the demand function for capital by a good firm is the minimum between the unconstrained demand for capital and the revelation borrowing limit:

$$k_g\left(r,\lambda\right) = \min\left\{k^{rv}\left(r,\lambda\right), k_g^*\left(r\right)\right\}$$

[12]Note that when the non-default allocation is binding then $\theta_b R'\left(k_b^{nd}\left(r,\lambda\right)\right) > (1+r)$.

Thus, the demand for capital by a good firm is decreasing in the interest rate r and increasing in the degree of law enforceability λ when the non-default borrowing limit is binding. The profits of good firms are:

$$\pi_g(r,\lambda) = \max_{k\in[0,k^{rv}(r,\lambda)]} \theta_g R(k) - (1+r)k = \theta_g R(k_g(r,\lambda)) - (1+r)k_g(r,\lambda)$$

The profits of good firms are decreasing in r and increasing λ, being strictly increasing in λ when the revelation constraint is binding.

We know from section 4 (proposition 13) that the revelation constraint, which affects good firms, is binding for a level of λ at which the non-default constraint of bad firms is still not binding. Thus, if we depart from the case of perfect law enforcement, $\lambda = 1$, and the degree of law enforceability drops enough, we will reach a point in which the revelation constraint would start to be binding for the good type, while the non default constraint is not binding yet for the bad type. When this happens, the equilibrium allocation will not coincide any more with the efficient allocation. We define λ^{rvE} as the threshold degree of law enforceability at which the revelation constraint is satisfied with equality for the optimal capital of firms $\left(k_g^{*E}, k_b^{*E}\right)$ when the interest rate at equilibrium is equal to the one that occurs when law enforcement is perfect, r^*:

$$\lambda^{rvE} \overset{Def}{\Leftrightarrow} \theta_b R(k_b^{*E}) - (1+r^*)k_b^{*E} = (1-\lambda^{rvE})\theta_b R(k_g^{*E})$$

Note that if λ is below λ^{rvE} the revelation constraint does not hold for the optimal capital of firms $\left(k_g^{*E}, k_b^{*E}\right)$. Consequently, the equilibrium will not coincide with the case of perfect enforcement and the allocation will not be any more the efficient one. When λ is equal or above λ^{rvE}, neither the revelation constraint nor the non-default constraint are binding, and consequently the equilibrium will be as in the case of perfect law enforcement (the allocation will be the efficient one and the interest rate will be equal to r^*).

We define λ_b^{ndE} as the threshold level of λ such that the non default constraint of bad type (7) holds with equality at equilibrium for the unconstrained demand of capital for bad firms. If λ is smaller than λ_b^{ndE}, the non default constraint is binding for the bad type firm and the capital used for her will not be anymore her unconstrained demand. If λ is larger than λ_b^{ndE}, the non default constraint is not binding for the bad type firm and the capital used for her is equal to her unconstrained demand for capital. More precisely, λ_b^{ndE} is defined as the solution, together with a value of r, of the following system of equations[13]:

$$\theta_b R(k_b(r,\lambda)) - (1+r)k_b(r) = \left(1-\lambda_b^{ndE}\right)\theta_b R(k_b(r,\lambda)) \tag{35}$$

$$\nu\,\pi_g(r,\lambda) + (1-\nu)\,\pi_b(r,\lambda) - (1+r)k = 0 \tag{36}$$

where the first equation is the non-default constraint of bad type (7) holding with equality for the unconstrained demand for capital of bad type firm. The second equation is the arbitrage condition (21), which determines the equilibrium interest rate.

[13]Unlike the case of λ^{rvE}, when law enforcement is at the level of λ_b^{ndE} the interest rate is not anymore r^* and needs to be determined jointly with λ_b^{ndE}.

Remark 23 *Note that* λ^{rvE} *and* λ_b^{ndE} *are different from the definitions of* $\lambda^{rv}(r)$ *and* $\lambda_b^{nd}(r)$ *in section 4, where we used partial equilibrium. In the previous sections, both* λ^{rv} *and* λ_b^{nd} *were defined for a given interest rate (r was an exogenous variable). Thus,* λ^{rv} *and* λ_b^{nd} *in previous sections were functions of the interest rate. In this section, since the interest rate is an endogenous variable,* λ^{rvE} *and* λ_b^{ndE} *are not any more functions of the interest rate.*

Lemma 24 *There exists a unique threshold value of* λ*, denoted by* $\lambda_b^{ndE} < \lambda^{rvE}$*, such that the non-default constraint of bad type (7) holds with equality at equilibrium when* $\lambda = \lambda_b^{ndE}$*. When* $\lambda > \lambda_b^{ndE}$ *the non default constraint of bad type (7) is not binding, and when* $\lambda < \lambda_b^{ndE}$ *it is binding.*

Thus, it follows from lemma 24 and discussion above that: *i*) when $\lambda \geq \lambda^{rvE}$, incentive constraints are not binding and the market allocation is the same as with perfect law enforcement; *ii*) when $\lambda \in \left[\lambda^{ndE}, \lambda^{rvE}\right)$, the revelation constraint is binding for good firms while non-default constraints are not binding; *iii*) when $\lambda \in \left(0, \lambda^{ndE}\right)$, the revelation constraint is binding for the good type and the non-default constraint is binding for the bad type and the amount of capital used in both firms are the same (see lemma 15).

Proposition 25 *When* $\lambda \in \left(0, \lambda^{rvE}\right)$*, the capital of good firms* k_g *at the equilibrium increase with the degree of law enforceability* λ*. For the bad firms, there is* $\widetilde{\lambda} < \lambda^{ndE}$*, such that if* $\lambda \in \left(\widetilde{\lambda}, \lambda^{rvE}\right)$ *then* $k_b > k_b^{*E}$*. Furthermore, if* $\lambda \in \left(\lambda_b^{ndE}, \lambda^{rvE}\right)$*,* k_b *is a decreasing function of* λ *(being* $k_b = k_b^{*E}$ *when* $\lambda = \lambda^{rvE}$*). If* $\lambda \in \left(0, \lambda^{ndE}\right)$*,* k_b *is an increasing function of* λ*.*

The above proposition says that when law enforcement is imperfect, the size of good firms is inefficiently small. Furthermore, the size of bad firms may be inefficiently large. That is, the amount of capital allocated to bad firms is above its optimal level. The intuition of the above proposition is that when law enforcement is imperfect, good firms are constrained, and so is their demand for funds. A lower demand for funds by good firms implies lower interest rate (see proposition 27 below), which implies that bad firms (when not constrained) demand more capital than the optimal level k_b^{*E}. When law enforcement improves, good firms are less constrained and their demand for funds is higher, increasing the equilibrium interest rate, and reducing the demand for funds of bad firms. Thus, an improvement in law enforcement will make good firms grow, while the evolution of bad firms would not be monotonic (their size will grow for low levels of λ, exceeding their optimal size). If law enforcement keeps rising, the size of bad firms eventually starts shrinking, becoming closer to their optimal size.

Proposition 26 *When* $\lambda \in \left(0, \lambda^{rvE}\right)$*, the average firm size and the share of the capital that goes to good firms are inefficiently small:* $\nu k_g + (1-\nu) k_b < \nu k_g^{*E} + (1-\nu) k_b^{*E}$*;* $\frac{\nu k_g}{\nu k_g + (1-\nu) k_b} < \frac{\nu k_g^{*E}}{\nu k_g^{*E} + (1-\nu) k_b^{*E}}$*.*

Thus, propositions 25 and 26 show that when law enforcement is weak ($\lambda < \lambda^{rvE}$), firm size distribution is inefficient in three aspects: i) the size of the good type firm is always inefficiently small, while the size of the bad type firm is also inefficient, but it may be inefficiently small when λ is small enough ($\lambda < \widetilde{\lambda}$) or even inefficiently large, when λ is not too small ($\lambda > \widetilde{\lambda}$); ii) the average firm size is inefficiently small; iii) the share of the capital that goes to good firms is inefficiently small also. When law enforcement improves, incentive constraints loosen, and the firm size distribution becomes closer to the efficient firm size distribution in these three aspects. First, good firms grow and the effect on the firm size of bad firms is not monotonic, but eventually these firms will also get closer to their optimal level. Second, on average firm size increases, becoming closer to the efficient level. Third, this positive effect benefits specially the good firms, which suffer a tighter incentive constraint and consequently the share of the capital that goes to them rises, getting closer to its efficient level. Thus, the improvement of law enforcement contributes to make the firm size distribution closer to the efficient one. As a corollary, these effects of law enforceability on firm size distribution involve also effects in per capita income, as the following proposition shows.

Proposition 27 *When $\lambda \in \left(0, \lambda^{rvE}\right)$, the per capita income y, the average productivity y/k and the interest rate r increase with the degree of law enforceability λ.*

The mechanism behind these results is quite intuitive: when law enforcement improves, the incentive constraints and borrowing limits of firms are relaxed, and firms may choose on average a larger and more productive size. This positive effect affects especially the good firms, which in some sense have tighter incentive constraints than the bad firms (since the revelation constraint affecting good firms is tighter that the non-default constraint affecting bad firms). Thus, better law enforcement implies a more efficient firm size and a higher allocation of capital to the more productive firms, which entail higher productivity, per capita income and profits. The larger expected profits imply at equilibrium higher interest rates.

The results of this section are summarized in the following table:

7. Extension:

7.1. The Role of Own Capital and Collateral

Another interesting extension is to consider that each firm type has a certain amount of own capital a. We assume that $a < k_b^*$, which implies that the own capital is not enough to finance the optimal capital of firms, hence external finance is needed. This extension of the model would not affect any of the conclusions of the benchmark model. The interesting thing about it is the effect that the own capital and the collateral would have on the equilibrium allocation of capital. The basic effect of own capital and collateral is to ease the incentive constraints: given that the firm debt is smaller (due to own capital) and the remaining part of the capital is used as a collateral, it becomes more likely that the recoverable fraction of revenues of the firm plus the collateral exceed the debt of the firm, providing incentives for the firm to fulfill the contract. As in the previous subsection, we assume that the portion $(1 - \delta)$ of the capital of the firm remains after production and the fraction δ is depreciated. This remaining part of the capital will be pledged as collateral.

The non-default constraint when there is own capital and the remaining part of the capital can be used as a collateral would be as follows:

$$\theta_j\, R\,(k_j) + (1-\delta)k_j - (1+r)\,(k_j - a)\; \geq\; (1-\lambda)\theta_j\, R\,(k_j) \quad \forall j\, \in\, \{g,b\} \Leftrightarrow \tag{37}$$

$$(\delta + r)k_j - (1+r)a\; \leq\; \lambda\theta_j\, R\,(k_j) \quad \forall j\, \in\, \{g,b\} \Leftrightarrow \tag{38}$$

Thus, the non-default constraint is similar to the one in the benchmark model, there are only two differences: i) for a given amount of capital used by the firm, k_j, the amount of the debt of the firm, $(1+r)\,(k_j - a)$, is smaller since the firm uses also own capital; ii) furthermore, if the firm defaults, she loses the collateral, $(1-\delta)k_j$. Both the existence of own capital and the use of collateral increase the incentives of firms to fulfill the contract. The non-default borrowing limit is an increasing function of the own capital and a decreasing function of the part of the capital that cannot be used as a collateral: the depreciation:

$$k_j^{nd}\,(r,\lambda,a,\delta) \overset{Def}{\Leftrightarrow} \lambda\theta_j \frac{R\left(k_j^{nd}\,(r,\lambda)\right)}{k_j^{nd}\,(r,\lambda)} = (\delta + r) - (1+r)\frac{a}{k_j^{nd}\,(r,\lambda)} \tag{39}$$

$$\frac{\partial k_j^{nd}\,(r,\lambda,a,\delta)}{\partial a} = -\frac{(1+r)\frac{1}{k_j^{nd}}}{\lambda\theta_j \frac{\partial\left(\frac{R\left(k_j^{nd}\right)}{k_j^{nd}}\right)}{\partial k} - (1+r)\frac{a}{\left(k_j^{nd}\right)^2}} > 0 \tag{40}$$

$$\frac{\partial k_j^{nd}\,(r,\lambda,a,\delta)}{\partial \delta} = \frac{1}{\lambda\theta_j \frac{\partial\left(\frac{R\left(k_j^{nd}\right)}{k_j^{nd}}\right)}{\partial k} - (\delta+r)\frac{a}{\left(k_j^{nd}\right)^2}} < 0 \tag{41}$$

The revelation constraint would be as follows:

$$\pi_b + (1-\delta)k + (1+r)a\; \geq\; (1-\lambda)\theta_b R(k_g) \tag{42}$$

where π_b is defined similarly as in the rest of the paper:

$$\pi_b(r,\lambda,a,\delta) = \max_{k\in\left[0,k_b^{nd}(r,\lambda,a,\delta)\right]} \theta_b R\,(k) - (1+r)k$$

Thus, the revelation borrowing limit would be:

$$k^{rv}\,(r,\lambda,a,\delta)\; =\; \overset{def}{\Leftrightarrow} (1-\lambda)\theta_b R\,(k^{rv}\,(r,\lambda,a))\; =\; \pi_b\,(r,\lambda,a,\delta) + (1-\delta)k + (1+r)a \tag{43}$$

$$\frac{\partial k_j^{nd}\,(r,\lambda,a)}{\partial a} = \frac{\frac{\partial \pi_b(r,\lambda,a)}{\partial a} + 1}{(1-\lambda)\theta_b \frac{\partial\left(R\left(k_j^{nd}\right)\right)}{\partial k} - (1-\delta)} > 0 \tag{44}$$

$$\frac{\partial k_j^{nd}\,(r,\lambda,a)}{\partial \delta} = \frac{\frac{\partial \pi_b(r,\lambda,a)}{\partial \delta} - k}{(1-\lambda)\theta_b \frac{\partial\left(R\left(k_j^{nd}\right)\right)}{\partial k} - (1-\delta)} < 0 \tag{45}$$

Given that if the bad type firm decides not to truly reveal her type it's because she will default, and the gains from default decrease with the own capital and with the collateral, it follows that when bad firms have own capital, a, they have more incentives to truly reveal their type, and this makes the revelation borrowing limit higher. At the same time, the higher the depreciation, the lower is the part of capital that can be used as collateral, and hence the more incentives bad firms have to not reveal their type and default. Thus, the existence of own capital and the use of collateral ease the incentive constraints expanding both the non-default and the revelation borrowing limits.

8. Conclusions

Empirical evidence supports the idea that law enforcement plays an important role in determining the performance of the financial system and the firm size distribution. This paper has analyzed how the degree of law enforceability affects the way in which the financial system allocates capital to firms of different productivity levels and variable size, in a framework of imperfect enforcement and asymmetric information. Entrepreneurs use credit to fund a project, but they have different productivity (ability) which is known only to them, and this generates an adverse selection problem. Entrepreneurs can default and financial intermediaries can recover only a fraction of their revenue. We interpret this recoverable fraction as the degree of law enforceability of our economy. The imperfect enforceability of contracts and the asymmetric information imply that financial contracts should satisfy two types of incentive constraints: i) the non-default constraint, which means that firms should have incentive to repay their debts; and ii) the revelation constraint, which means that firms should truly reveal their type. These incentive constraints entail borrowing limits for firms and affect the allocation of capital to different firms by the financial system. Capital is not efficiently allocated because of two main reasons. Firstly, firm size is inefficiently small in the sense that the amount of capital that they use is below the optimal level. Secondly, the lenders do not always allocate efficiently the capital to firms of different productivity, as they cannot distinguish among borrowers. In fact, the good firms suffer tighter constraints than bad firms, since their level of borrowing is limited by the revelation constraint, which we showed that it is always more restrictive than the non-default constraint experienced by the bad firms.

These problems are worsened with imperfect law enforcement. There are tree types of equilibrium depending on the degree of law enforceability: i) When the law enforcement is effective enough, but not necessarily perfect, the incentive constrains are not binding and the allocation of capital to firms is efficient. ii) When the law enforcement is in a certain middle interval, incentive constraints are binding just for the good firms, which are underfinanced in the sense that get less capital than their optimal level, while incentive constraint of bad firms are not binding. Thus, the size of good firms is inefficiently small while the size of bad firms is optimal in a partial equilibrium framework while it is inefficiently large in general equilibrium. iii) Finally, when the law enforcement is weaker than a certain threshold, both good and bad firms are constrained. It turns out that in this type of equilibrium both firm types receive the same amount of capital, which is inefficiently small in the sense that is smaller than their optimal level. Thus, more effective law enforcement eases incentive constraints and, as a consequence, the firm size distribution improves with law enforcement, getting closer to the optimal firm size distribution. This implies that per capita output, productivity and lenders interest rate all increase with law enforcement.

We have also extended our model to include own capital and the use of capital as a collateral and the main results of the benchmark model do not change with these modifications.

Thus, we have constructed a theory that links the degree of law enforceability with the functioning of the financial system, the firm size distribution and the productivity. Information asymmetries and the heterogeneity in firms' productivity generate an adverse selection problem, which plays a crucial role in our theory.

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9. Appendix

Proof Lemma 5:

Let be $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ the equilibrium menu of contracts and assume that there is $j \in \{g, b\}$ such that $\chi_j(1+\widehat{i}_j)\widehat{k}_j + (1\text{-}\chi_j)\left[(\lambda\text{-}\psi)\theta_j R(\widehat{k}_j)\right]\text{-}(1+r)\widehat{k}_j > 0$. There are two cases:

1 If firm j defaults at equilibrium:

$$\lambda\theta_j R(\widehat{k}_j) > (1+\widehat{i}_j)\widehat{k}_j \Rightarrow \tag{46}$$

$$\left[(\lambda\text{-}\psi)\theta_j R(\widehat{k}_j)\right] > (1+r)\widehat{k}_j \Rightarrow \lambda\theta_j R(\widehat{k}_j) > (1+r)\widehat{k}_j + \psi\theta_j R(\widehat{k}_j) \Rightarrow \tag{47}$$

$$\widehat{\pi}_j = (1-\lambda)\theta_j R(\widehat{k}_j) < \theta_j R(\widehat{k}_j) - (1+r)\widehat{k}_j - \psi\theta_j R(\widehat{k}_j) \tag{48}$$

Consider the contract $(\widetilde{i}_j, \widetilde{k}_j)$ such that $\widetilde{k}_j = \widehat{k}_j$ and $1 + \widetilde{i}_j = (1+r) + \psi\theta_j R(\widehat{k}_j)$. It follows from (47) and (48) that firm j does not default with the contract $(\widetilde{i}_j, \widetilde{k}_j)$ and has more profits with the contract $(\widehat{i}_j, \widehat{k}_j)$ than at equilibrium. Obviously the financial intermediary has positive profits with the contract $(\widetilde{i}_j, \widetilde{k}_j)$. Furthermore, given that the revelation constraint for type $j' \neq j$ is satisfied at equilibrium:

$$\widehat{\pi}_{j'} \geq \max\left\{\theta_{j'} R(\widehat{k}_j) - (1+\widehat{i}_j)\widehat{k}_j, (1-\lambda)\theta_{j'} R(\widehat{k}_j)\right\} \geq (1-\lambda)\theta_{j'} R(\widehat{k}_j) \tag{49}$$

Thus:

- If the revelation constraint holds for the type $j' \neq j$ with the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then in the contract $(\widetilde{i}_j, \widetilde{k}_j)$ firm j is better off than in the equilibrium contract, financial intermediary has positive profit and the revelation constraint holds for $j' \neq j$. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ is not an equilibrium contract $\Rightarrow\Leftarrow$

- If the revelation constraint does not hold for the type j' for the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then using (49) it follows that:

$$(1-\lambda)\theta_{j'} R(\widehat{k}_j) \leq \max\left\{\theta_{j'} R(\widehat{k}_j) - (1+\widehat{i}_j)\widehat{k}_j, (1-\lambda)\theta_{j'} R(\widehat{k}_j)\right\} \leq \widehat{\pi}_{j'}$$
$$< \theta_{j'} R(\widehat{k}_j) - (1+\widetilde{i}_j)\widehat{k}_j$$

  Thus, if a financial intermediary offers the contract $(\widetilde{i}_j, \widetilde{k}_j)$ to both types, according to the above equation, type $j\prime$ would not default and would be better off than at equilibrium, exactly the same happens for type j and the financial intermediary would have positive profits. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ is not an equilibrium menu of contracts $\Rightarrow\Leftarrow$

2 If firm j does not default at equilibrium. Then the non-default constraint and the positive profits of financial intermediary imply that:

$$\lambda\theta_j R(\widehat{k}_j) \leq (1+\widehat{i}_j)\widehat{k}_j < (1+r)\widehat{k}_j \tag{50}$$

Consider the contract $(\widetilde{i}_j, \widetilde{k}_j) = (r, \widehat{k}_j)$. It follows from the above equation that at such contract the non-default constraint holds for type j, financial intermediary has zero profits and firms of type j are better off. Thus:

- If the revelation constraint holds for the type j' for the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then in the contract $(\widetilde{i}_j, \widetilde{k}_j)$ the financial intermediaries has zero profits and firms of type j are better off than at the equilibrium contract. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b)$ is not an equilibrium contract. $\Rightarrow\Leftarrow$

- If the revelation constraint does not hold for the type j' for the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then using (49) it follows that:

$$(1-\lambda)\theta_{j'}R(\widehat{k}_j) \leq \max\left\{\theta_{j'}R(\widehat{k}_j) - (1+\widehat{i}_j)\widehat{k}_j, (1-\lambda)\theta_{j'}R(\widehat{k}_j)\right\} \leq \widehat{\pi}_{j'}$$
$$< \theta_{j'}R(\widehat{k}_j) - (1+\widetilde{i}_j)\widehat{k}_j$$

  Thus, if a financial intermediary offers the contract $(\widetilde{i}_j, \widetilde{k}_j)$ to both types, according to the above equation type $j\prime$ would not default and would be better off than at equilibrium, exactly the same happens for type j and the financial intermediary would have zero profits. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ is not an equilibrium menu of contracts. $\Rightarrow\Leftarrow$ ■

Proof lemma 7

Let's be $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ the equilibrium menu of contracts and assume that there is a type of firm $j \in \{g, b\}$ that defaults at equilibrium. Thus, it follows from lemma 5 that:

$$\left[(\lambda\text{-}\psi)\theta_j R(\widehat{k}_j)\right] = (1{+}r)\widehat{k}_j \Rightarrow \lambda\theta_j R(\widehat{k}_j) = (1{+}r)\widehat{k}_j + \psi\theta_j R(\widehat{k}_j) \tag{51}$$

Consider the contract $(\widetilde{i}_j, \widetilde{k}_j) = (r, \widehat{k}_j)$. It follows from the above equation that in such contract the non-default constraint holds and firms of type j are better off:

$$\lambda\theta_j R(\widehat{k}_j) = (1{+}r)\widehat{k}_j + \psi\theta_j R(\widehat{k}_j) > (1{+}r)\widehat{k}_j$$
$$\widehat{\pi}_j = \theta_j R(\widehat{k}_j) - (1{+}r)\widehat{k}_j - \psi\theta_j R(\widehat{k}_j) < \theta_j R(\widehat{k}_j) - (1+r)\widehat{k}_j = \widetilde{\pi}_j$$

Thus:

- If the revelation constraint holds for the type j' for the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then in the contract $(\widetilde{i}_j, \widetilde{k}_j)$ the financial intermediaries have zero profits and firms of type j are better off than at the equilibrium contract. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b)$ is not an equilibrium contract. $\Rightarrow\Leftarrow$

- If the revelation constraint does not hold for the type j' for the contract $(\widetilde{i}_j, \widetilde{k}_j)$, then using (49) it follows that:

$$(1-\lambda)\theta_{j'}R(\widehat{k}_j) \leq \max\left\{\theta_{j'}R(\widehat{k}_j) - (1+\widehat{i}_j)\widehat{k}_j, (1-\lambda)\theta_{j'}R(\widehat{k}_j)\right\} \leq \widehat{\pi}_{j'}$$
$$< \theta_{j'}R(\widehat{k}_j) - (1+\widetilde{i}_j)\widehat{k}_j$$

Thus, if a financial intermediary offers the contract $(\widetilde{i}_j, \widetilde{k}_j)$ to both types, according to the above equation type $j\prime$ would not default and would be better off than at equilibrium, exactly the same happens for type j and the financial intermediary would have zero profits. Thus, $(\widehat{i}_g, \widehat{k}_g, \widehat{i}_b, \widehat{k}_b,)$ is not an equilibrium menu of contracts. $\Rightarrow\Leftarrow$ ■

Proof lemma 9:
There are two cases:
1. If the non-default constraint is not binding for the bad type, then:

$$\left.\begin{array}{l} \theta_b R(k_b) - (1+r)k_b = \max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\} \\ \theta_b R(k_b) - (1+r)k_b = \max_k \theta_b R(k) - (1+r)k \geq \theta_b R(k_g) - (1+r)k_g \end{array}\right\} \Rightarrow$$

$$\max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\} \geq \theta_b R(k_g) - (1+r)k_g \Rightarrow$$
$$\max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\} = (1-\lambda)\theta_b R(k_g)$$

2. If the non-default constraint is binding for the bad type: then it follows from the non-default constraint and the fact that the revenue function $R(k)$ is increasing that $\forall k_g > k_b$ the revelation constraint does not hold:

$$\forall k_g > k_b \quad \theta_b R(k_b) - (1+r)k_b = (1-\lambda)\theta_b R(k_b) < (1-\lambda)\theta_b R(k_g)$$
$$\leq \max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\}$$

Thus, in this case $k_g = k_b$. Since the non-default constraint is binding for the bad type:

$$\left.\begin{array}{l} \theta_b R(k_b) - (1+r)k_b = (1-\lambda)\theta_b R(k_b) \\ k_g = k_b \end{array}\right\} \Rightarrow$$

$$\theta_b R(k_g) - (1+r)k_g = (1-\lambda)\theta_b R(k_g) \Rightarrow$$
$$\max\{\theta_b R(k_g) - (1+r)k_g, (1-\lambda)\theta_b R(k_g)\} = (1-\lambda)\theta_b R(k_g)$$

■

Proof Lemma 10:
If the non-default constraint is binding for the good type then $k_g^* > k_g^{nd}$, which implies that the profits are increasing in the capital at the non-default borrowing limit:

$$\frac{\partial \pi}{\partial k} = \theta_g R'(k_g^{nd}) - (1+r) > \theta_g R'(k_g^*) - (1+r) = 0$$

Using the definition of non-default borrowing limit:

$$(1-\lambda)\theta_g R(k_g^{nd}) = \theta_g R(k_g^{nd}) - (1+r)k_g^{nd} \Rightarrow$$

$$(1-\lambda)\theta_b R(k_g^{nd}) = \frac{\theta_b}{\theta_g}\left[\theta_g R(k_g^{nd}) - (1+r)k_g^{nd}\right] > \frac{\theta_b}{\theta_g}\left[\theta_g R(k_b^{nd}) - (1+r)k_b^{nd}\right] >$$
$$\frac{\theta_b}{\theta_g}\left[\theta_g R(k_b) - (1+r)k_b\right] = \theta_b R(k_b) - (1+r)\frac{\theta_b}{\theta_g}k_b > \theta_b R(k_b) - (1+r)k_b$$

■

Proof Lemma 15:

The revelation constraint in the case that $k_b = k_b^{nd}$ is as follows:

$$\theta_b R(k_b^{nd}) - (1+r)k_b^{nd} \geq (1-\lambda)\theta_b R(k_g)$$

Take any $k_g > k_b^{nd}$, it follows from the definition of k_b^{nd} that:

$$\theta_b R(k_b^{nd}) - (1+r)k_b^{nd} = (1-\lambda)\theta_b R(k_b^{nd}) < (1-\lambda)\theta_b R(k_g)$$

■

Proof Lemma 19

Assume that the bad type firm fulfills the contract $\theta_b R(k) - (1+i)k \geq (1-\lambda)\theta_b R(k)$ $\Leftrightarrow (1+i)k \leq \lambda\theta_b R(k) < \lambda\theta_g R(k)$, then the good type firm does not default either. Then the zero profit condition of financial intermediary implies that i is equal to r:

$$\nu\,[(1+i)k\text{-}(1+r)k] + (1\text{-}\nu)\,[(1+i)k\text{-}(1+r)k] = (i-r)k \;=\; 0 \Rightarrow i = r$$

Therefore, the zero profit condition of the financial intermediary holds separately for the two firms type, therefore the contract is not a pooling contract according with our definition (see definition 4). ■

Proof Lemma 20

Consider that the good type defaults in equilibrium $\theta_g\, R\,(k) - (1+i)k \;<\; (1-\lambda)\theta_g\, R\,(k)$. Then, this implies that the bad type defaults also

$$\theta_g\, R\,(k) - (1+i)k \;<\; (1-\lambda)\theta_g\, R\,(k) \Leftrightarrow (1+i)k \;>\; \lambda\theta_g\, R\,(k) \;\;>\; \lambda\theta_b\, R\,(k)$$

Since both firm types default, the zero profit condition of financial intermediary would be as follows:

$$\nu\,[(\lambda\text{-}\psi)\theta_g R(k)\text{-}(1+r)k] + (1\text{-}\nu)\,[(\lambda\text{-}\psi)\theta_b R(k)\text{-}(1+r)k] \;=\; 0 \tag{52}$$

Note that $\theta_g \;>\; \theta_b \Rightarrow$

$$(\lambda-\psi)\theta_g R(k) - (1+r)k \;>\; (\lambda-\psi)\theta_g R(k) - (1+r)k$$

Thus, the above two equations implies that the good firms are cross-subsidizing the bad firms:

$$(\lambda-\psi)\theta_g R(k) - (1+r)k \;>\; 0 \;>\; (\lambda-\psi)\theta_b R(k) - (1+r)k$$

Consider another pooling contract (k', i') defined as follows: $k' = k$ and

$$i' \overset{\text{Definition}}{\Leftrightarrow} \nu\,[((1+i')k\text{-}(1+r)k] + (1-\nu)\,[(\lambda\text{-}\psi)\theta_b R(k)\text{-}(1+r)k] \;=\; 0 \Leftrightarrow$$
$$(1+i')k \;=\; (1+r)k + \frac{1-\nu}{\nu}\,[(1+r)k - (\lambda\text{-}\psi)\theta_b R(k)]$$

Note also that the zero profit condition of the financial intermediary (52) implies that:

$$\lambda\theta_g R(k) \;=\; [\psi\theta_g R(k) + (1+r)k] + \frac{(1\text{-}\nu)}{\nu}\,[(1+r)k - (\lambda\text{-}\psi)\theta_b R(k)]$$
$$= \psi\theta_g R(k) + (1+i')k \;>\; (1+i')k$$

Thus, in the contract (i', k') the good firm type does not default. Furthermore, using the above equation it follows that the good firm type is better off:

$$R(k') - (1+i')k' = R(k) - (1+i')k > \lambda\theta_g R(k)$$

The bad type firm is exactly as in the initial contract if $\lambda\theta_g R(k) \leq (1+i')k$, and it is better of if $\lambda\theta_g R(k) > (1+i')k$. ■

Proof proposition 21

The constraints in the pooling contract are the following:

$$\begin{aligned} \theta_g R(k) - (1+i)k &\geq [1-\lambda\mu]\,\theta_g R(k) \\ (1+r)k &= (1-\nu)\,(\lambda-\psi)\,\theta_b R(k)\mu + \nu(1+i)k \end{aligned}$$

These two constraints imply that:

$$\begin{aligned} -\nu(1+i)k &\geq -\nu\lambda\mu\theta_g R(k) \\ -(1+r)k + \nu(1+i)k &= -(1-\nu)\,(\lambda-\psi)\,\theta_b R(k)\mu \\ -(1+r)k &\geq -\mu\left[\lambda\left(\nu\theta_g + (1-\nu)\theta_b\right) - (1-\nu)\mu\psi\right] R(k) \end{aligned}$$

$$\begin{aligned} \lambda\mu\left[\left[\nu\theta_g + (1-\nu)\theta_b\right] - (1-\nu)\psi\theta_b\right] R(k) &\geq (1+r)k \\ (1+i)k &= \frac{(1+r)k - (1-\nu)\mu\left[\lambda-\psi\right]\theta_b R(k)}{\nu} \end{aligned}$$

Let's consider the maximization problem:

$$\begin{aligned} &\max_{k,\mu} \theta_g R(k) - \frac{(1+r)k + (1-\nu)\mu\,(\psi-\lambda)\,\theta_b R(k)}{\nu} \\ &s.t.\text{:}\ \left[\nu\lambda\theta_g + (1-\nu)(\lambda-\psi)\theta_b\right]\mu R(k) \geq (1+r)k \end{aligned}$$

Consider the case in which $\psi \geq \frac{\lambda^{rv}[\nu\theta_g+(1-\nu)\theta_b]}{(1-\nu)\theta_b}$. In this case the only feasible capital would be $k=0$ and then the profits of good firms would be equal to zero. Then, we define the profit functions for good firms in the case of pooling equilibrium:

$$\begin{aligned} \pi_g^{pooling}(\lambda,\psi) &= \max_{k,i,\mu} \theta_g R(k) - (1+i)k \\ &\quad s.t.\ \theta_g R(k) - (1+i)k \geq (1-\lambda)\mu\theta_g R(k) \\ &\qquad\quad (1+r)k \leq (1-\nu)\left[(\lambda-\psi)\,\theta_b R(k)\right]\mu + \nu(1+i)k \end{aligned} \tag{53}$$

We define the profit functions for good firms in the case of separating equilibrium:

$$\begin{aligned} \pi_b^{separating}(\lambda) = &\ \arg\max_k \theta_b R(k) - (1+r)k \\ &\quad s.t.\ \theta_b R(k) - (1+r)k \geq (1-\lambda)\theta_b R(k) \end{aligned} \tag{54}$$

$$\begin{aligned} \pi_g^{separating}(\lambda) = &\ \arg\max_k \theta_g R(k) - (1+r)k \\ &\quad s.t.\ \pi_b^{separating}(\lambda) \geq (1-\lambda)\theta_b R(k) \end{aligned} \tag{55}$$

We have seen that when $\psi \geq \frac{\lambda^{rv}[\nu\theta_g+(1-\nu)\theta_b]}{(1-\nu)\theta_b}$ and $\lambda \geq \lambda^{rv}$ $\pi_g^{pooling}(\lambda,\psi) = 0$ while $\forall\lambda > 0$ $\pi_g^{separating}(\lambda) > 0$. Then, it is possible to define $\underline{\psi}$ as follows:

$$\underline{\psi} = \max\left\{\inf\left\{\psi \in \Re \text{ s. t. } \sup_{\lambda\in[0,\lambda^{rv}]}\left\{\pi_g^{pooling}(\lambda,\psi) - \pi_b^{separating}(\lambda)\right\} \leq 0\right\}, 0\right\}$$

■

Proof Lemma 24

It follows from the definition of λ_b^{ndE} that when $\lambda = \lambda_b^{ndE}$ the non default constraint for the bad type holds with equality for the optimal capital of the bad type for the equilibrium interest rate. Furthermore, it follows from lemma 15 that the good and the bad type receive the same amount of capital. Thus, the following equations should hold when $\lambda = \lambda_b^{ndE}$:

$$\lambda_b^{ndE}\theta_b R\left(k_b^*(r)\right) - (1+r)k_b^*(r) = 0 \tag{56}$$
$$k_g = k_b = k_b^*(r) \tag{57}$$
$$\nu\left[\theta_g R\left(k_g\right) - (1+r)k_g\right] + (1-\nu)\left[\theta_b R\left(k_b\right) - (1+r)k_b\right] - (1+r)k = 0 \tag{58}$$

where the last equation is the equilibrium condition (21). Combining the two last equations we get:

$$\nu\left[\theta_g R\left(k_b^*(r)\right) - (1+r)k_b^*(r)\right] + (1-\nu)\left[\theta_b R\left(k_b^*(r)\right) - (1+r)k_b^*(r)\right] - (1+r)k = 0$$

Let´s define $\digamma(r)$ as the function in the left hand of the above equation:

$$\digamma(r) = \nu\left[\theta_g R\left(k_b^*(r)\right) - (1+r)k_b^*(r)\right] + (1-\nu)\left[\theta_b R\left(k_b^*(r)\right) - (1+r)k_b^*(r)\right] - (1+r)k$$

Note that:

$$\begin{aligned}
&\digamma(r^*) = \\
&\nu\left[\theta_g R\left(k_b^*(r^*)\right) - (1+r^*)k_b^*(r^*)\right] + (1-\nu)\left[\theta_b R\left(k_b^*(r^*)\right) - (1+r^*)k_b^*(r^*)\right] - (1+r^*)k < \\
&\nu\left[\theta_g R\left(k_g^*(r^*)\right) - (1+r^*)k_b^*(r^*)\right] + (1-\nu)\left[\theta_b R\left(k_b^*(r^*)\right) - (1+r^*)k_b^*(r^*)\right] - (1+r^*)k = 0
\end{aligned}$$

where we have used equation (33). Furthermore, $\lim\limits_{r\to-1}\digamma(r) > 0$. Thus, it follows from the Medium Value Theorem that there is a interest rate $\widetilde{r}$ such that $\digamma(\widetilde{r}) = 0$. Furthermore, $\digamma(r)$ is an increasing function:

$$\begin{aligned}
&\digamma'(r) = \left[\nu\left[\theta_g R'\left(k_b^*(r)\right) - (1+r)\right] + (1-\nu)\left[\theta_b R'\left(k_b^*(r)\right) - (1+r)\right]\right]\frac{\partial k_b^*(r)}{\partial r} = \\
&\nu\left(\theta_g - \theta_b\right)R'\left(k_b^*(r)\right)\frac{1}{-R"\left(k_b^*(r)\right)} > 0
\end{aligned}$$

where we have used the definition of $k_b^*(r)$ (see equation 1). Therefore there is a unique interest rate $\widetilde{r}$ such that $\digamma(\widetilde{r}) = 0$. Thus, there is a unique λ_b^{ndE} which satisfies equations (56) to (58), $\lambda_b^{ndE} = \frac{(1+\widetilde{r})k_b^*(\widetilde{r})}{\theta_b R\left(k_b^*(\widetilde{r})\right)} = \frac{R'\left(k_b^*(\widetilde{r})\right)k_b^*(\widetilde{r})}{R\left(k_b^*(\widetilde{r})\right)} = \varepsilon^R\left(k_b^*(\widetilde{r})\right)$. Proposition 27 establishes that the interest rate is a strictly increasing function of λ when $\lambda < \lambda^{rvE}$ (see proof below). Thus, when $\lambda > \lambda_b^{ndE}$ then $r(\lambda) > r\left(\lambda_b^{ndE}\right)$, and $F(r(\lambda)) < 0$ which implies that at equilibrium $k_g > k_b^*(r(\lambda))$. This implies according with lemma 15 that the non-default constraint of bad type is not binding. When $\lambda < \lambda_b^{ndE}$ then $r(\lambda) > r\left(\lambda_b^{ndE}\right)$, and $F(r(\lambda)) > 0$ which implies that at equilibrium $k_b > k_b^*(r(\lambda))$. Thus, in this case the non-default borrowing constraint is binding for the bad type. ■

Proof Proposition 27

Using the non arbitrage condition (21), it follows that

$$\nu\,\pi_g(r,\lambda) + (1-\nu)\,\pi_b(r,\lambda) - (1+r)k \;=\; 0$$
$$\frac{\partial r}{\partial \lambda} \;=\; \frac{\nu\,\frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial \lambda}}{k - \nu\,\frac{\partial \pi_g(r,\lambda)}{\partial r} - (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial r}} > 0$$

It is easy to check that $y = rk$:

$$y \;=\; n\left[\nu\,\theta_g R(k_g) + (1-\nu)\theta_b R(k_b)\right] - k \;=$$
$$n\left[\nu\,\theta_g \pi_g + (1-\nu)\theta_b \pi_b + (1+r)\left[\nu\,k_g + (1-\nu)k_b\right]\right] - k \;=$$
$$n\left[(1+r)k + (1+r)\frac{(1-n)}{n}k\right] - k \;=\; rk$$

Thus:

$$\frac{\partial y}{\partial \lambda} \;=\; \frac{\partial r}{\partial \lambda}k < 0; \qquad \frac{\partial (y/k)}{\partial \lambda} \;=\; \frac{\partial r}{\partial \lambda} < 0$$

■

Proof Proposition 26

The following system of equations determines the equilibrium interest rate and the per capita number of firms:

$$\nu\,\pi_g(r,\lambda) + (1-\nu)\,\pi_b(r,\lambda) - (1+r)k \;=\; 0$$
$$n\left[\nu\,k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)\right] - (1-n)k \;=\; 0$$

Differentiating the above system of equations:

$$\begin{bmatrix} \nu\,\frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial r} - k & 0 \\ n\left[\nu\,\frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial r}\right] & \left[\nu\,k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)\right] + k \end{bmatrix} \begin{bmatrix} dr \\ dn \end{bmatrix} =$$
$$-\begin{bmatrix} \nu\,\frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial \lambda} \\ n\left[\nu\,\frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\,\frac{\partial \pi_b(r,\lambda)}{\partial \lambda}\right] \end{bmatrix} d\lambda$$

Applying the Cramer's rule:

$$\frac{\partial r}{\partial \lambda} = -\frac{\begin{vmatrix} \nu \frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial \lambda} & 0 \\ n\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial \lambda}\right] & [\nu\, k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)] + k \end{vmatrix}}{\begin{vmatrix} \nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r} - k & 0 \\ n\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r}\right] & [\nu\, k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)] + k \end{vmatrix}} > 0$$

$$\left(\frac{\partial r}{\partial \lambda} = -\frac{\begin{vmatrix} \oplus & 0 \\ \oplus & \oplus \end{vmatrix}}{\begin{vmatrix} \ominus & 0 \\ \ominus & \oplus \end{vmatrix}} > 0 \right)$$

$$\frac{\partial n}{\partial \lambda} = -\frac{\begin{vmatrix} \nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r} - k & \nu \frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial \lambda} \\ n\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r}\right] & n\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial \lambda}\right] \end{vmatrix}}{\begin{vmatrix} \nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r} - k & 0 \\ n\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r}\right] & [\nu\, k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)] + k \end{vmatrix}} =$$

$$\frac{\left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial \lambda} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial \lambda}\right][-nk]}{\left[k - \left[\nu \frac{\partial \pi_g(r,\lambda)}{\partial r} + (1-\nu)\frac{\partial \pi_b(r,\lambda)}{\partial r}\right]\right][\nu\, k_g(r,\lambda) + (1-\nu)k_b(r,\lambda)] + k} < 0$$

Using capital market clearing condition (22) it follows that:

$$\nu k_g + (1-\nu)\, k_b = \left(\frac{1}{n} - 1\right) k < \left(\frac{1}{n^*} - 1\right) k = \nu k_g^{*E} + (1-\nu)\, k_b^{*E}$$

Thus, the average firm size, measured as the average amount of capital per firm, is below the optimal level. Furthermore, it was proven in section 4 that the share of the capital that goes to good firms is inefficiently small. ■

Proof Proposition 25

When $\lambda \in \left(\lambda_b^{ndE}, \lambda^{rvE}\right)$ $k_b = k_b^*(r)$. Since the .optimal level of capital $k_b^*(r)$ is strictly decreasing in the equilibrium interest rate and the interest rate is increasing in λ (proposition **27**), it follows that k_b is decreasing in λ:

$$\frac{\partial k_b}{\partial \lambda} = \underbrace{\frac{\partial k_b^*}{\partial r}}_{\ominus}\underbrace{\frac{\partial r}{\partial \lambda}}_{\oplus} < 0$$

When $\lambda = \lambda^{rvE}$ the equilibrium interest rate is r^*. Thus, when $\lambda \in \left(\lambda_b^{ndE}, \lambda^{rvE}\right)$, $r < r^*$(proposition **27**) $\Rightarrow k_b = k_b^*(r) > k_b^*(r^*) = k_b^{*E}$. ■

Figure 1. Non-Default Constraint

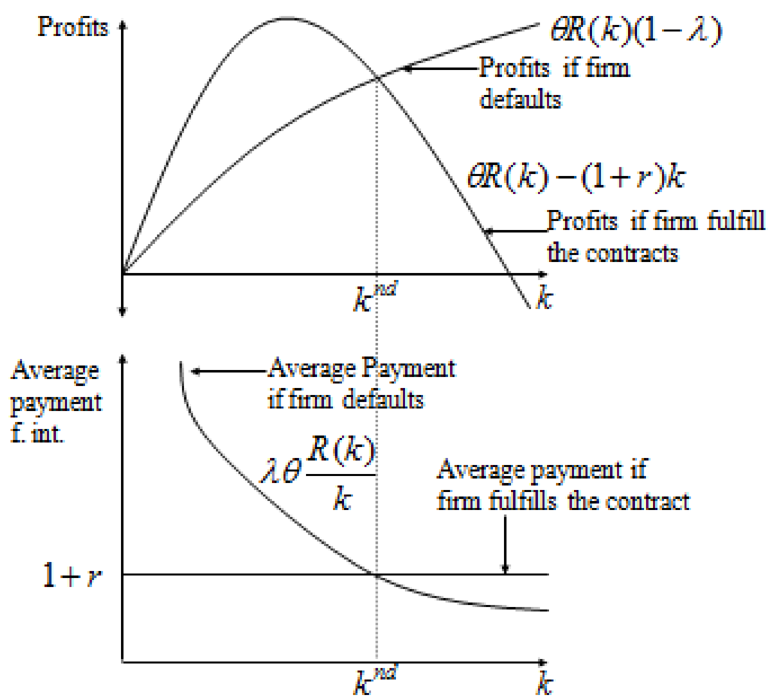


Figure 2. The Non-Default Borrowing Limit

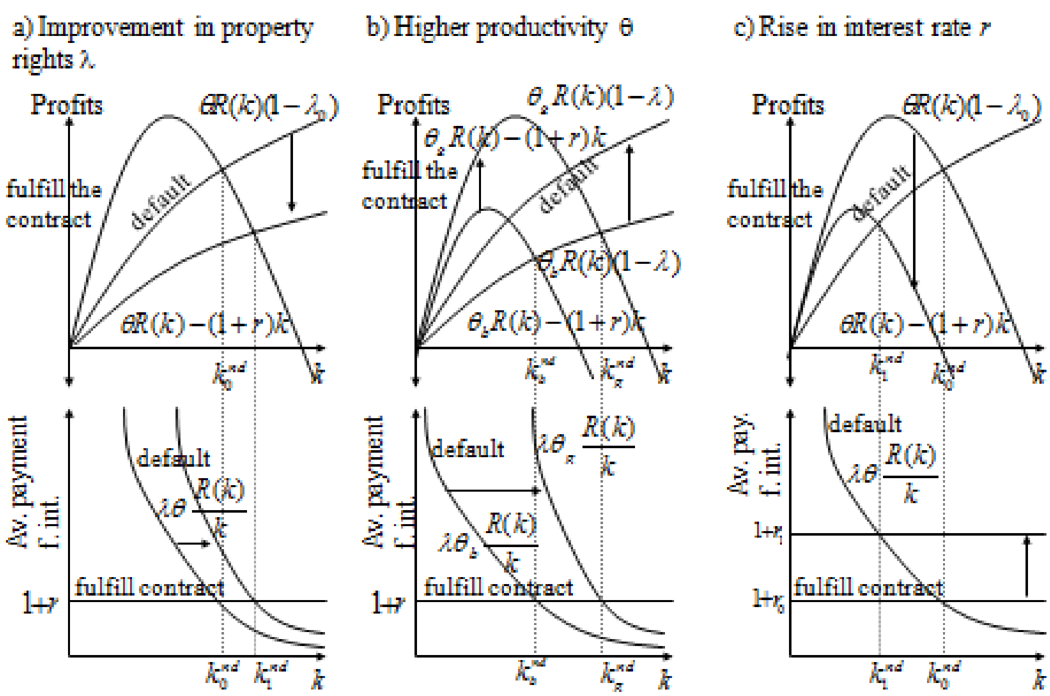


Figure 3. Incentive constraints are not binding: $\lambda > \lambda^{rv}$

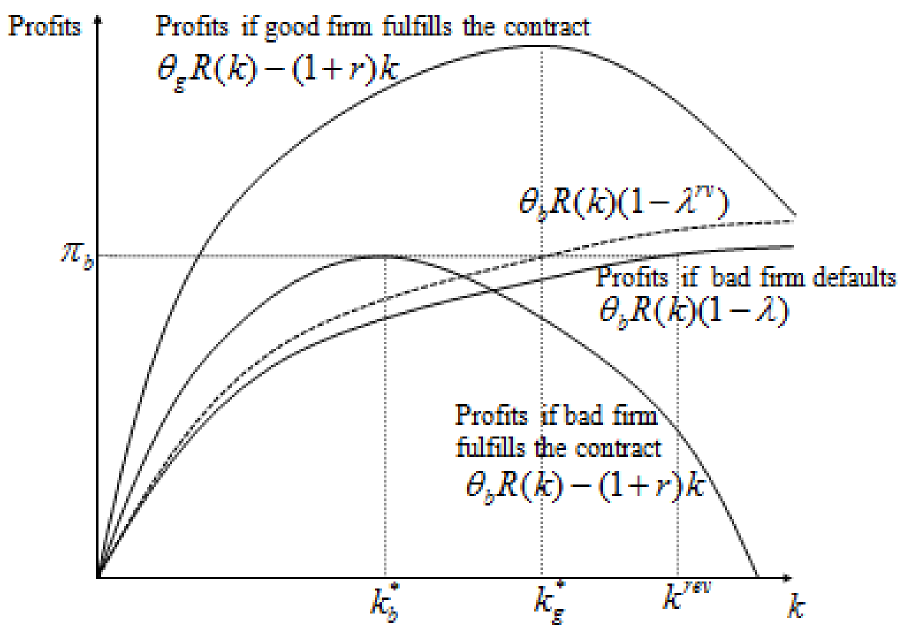


Figure 4. Revelation constraint is binding for the good type: $\lambda \in (\lambda_b^{nd}, \lambda^{rv})$

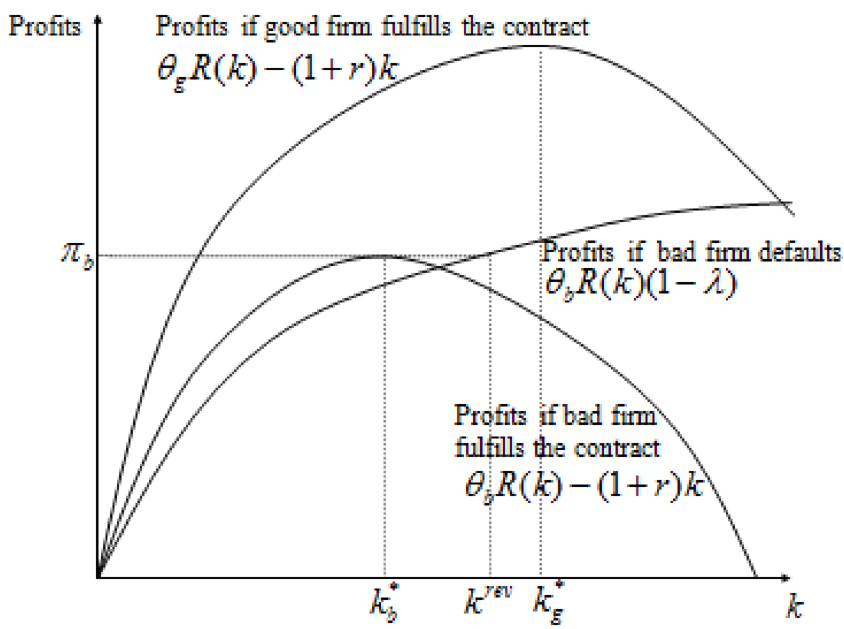
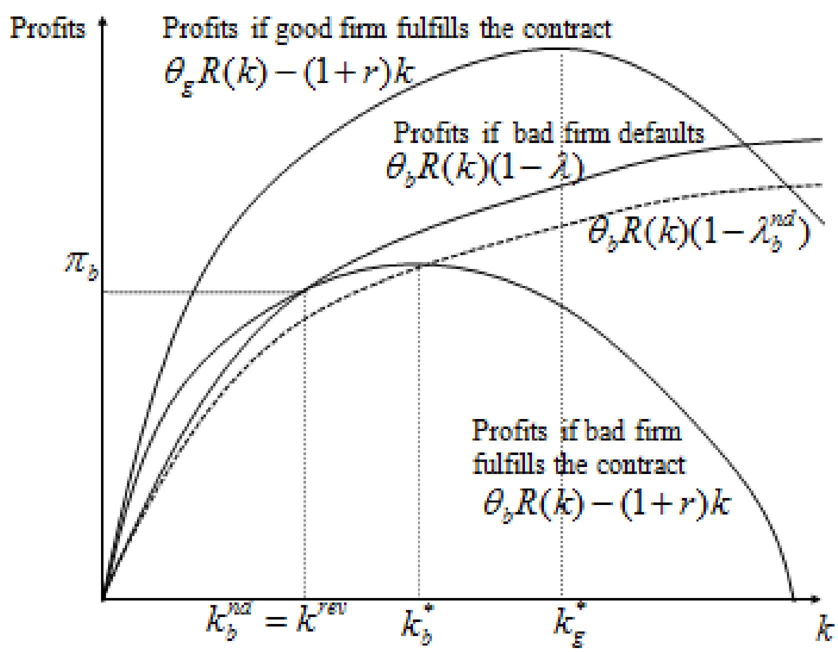


Figure 5. Revelation constraint is binding for the good type and non-default constraint is binding for the bad type: $\lambda < \lambda_b^{nd}$



Chapter 2

Aggregate Consequences of Adverse Selection*

March 18, 2014

Abstract

We develop a model where financial intermediation is affected by imperfect property rights and by adverse selection. These two frictions influence the factor allocation across firms which differ in their productivity and have endogenous size. The model allows us to quantify the effect of property rights and of adverse selection on output, firm size and productivity. While confirming the importance of property rights for development, our results suggest a significant role for adverse selection as well (with potential output gains of up to 20%).

1. Introduction

Many developing countries fail to achieve a successful development process (see Quah 1996, 1997 and Parente and Prescott, 1993)[1]. Explaining these differences in development has been one of the main themes of growth and development theory. While it is acknowledged that differences in physical and human capital per worker play an important role, several authors, like Prescott (1998) or Hall and

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[1] A worldwide statistical partnership, the International Comparison Program (ICP), collects official data that makes possible to compare the output of economies controlling for differences in price levels. Based on the ICP data, the Penn World Table (PWT) provides additional statistics which are widely used in the literature.

Jones (1999) have argued that differences in capital are not able to fully account for the differences in output in the data. This implies that not only the capital per worker, but also the TFP varies across countries. Therefore, to understand the huge differences in output per worker, we need to better understand not only the capital accumulation process, but also what factors lay behind TFP variation.

Among various possible explanations for the empirical fact considered, one recurrent theme in recent years has been the level of institutional development of countries, sometimes also called 'social infrastructure' (Hall and Jones 1999). While this term is quite broad, it is considered to encompass various aspects like the quality of property rights and the level of contract enforcement. In a seminal paper, La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997) documented that better law enforcement and accounting standards, positively impact on the development of financial markets. If we consider their finding together with the long tradition, going back to McKinnon (1973) up to Fry (1995) and King and Levine (1993), underlining that the development of the financial sector spurs economic growth, an empirical pattern emerges, where the quality of property rights enforcement stimulates financial development, and both contribute to the economic development. This pattern is consistent with the analysis of institutions as a fundamental cause of long term growth done by Acemoglu et all (2005), where they document with data and historical examples that the quality of property rights and the market imperfections affect development.

Empirical evidence suggesting that firm size is positively related to financial development and property rights protection (Beck, Demirgüĉ-Kunt and Maksimovic 2006) motivates us to pay particular attention to the role of firm size. It could be that a lower quality of property rights acted as a constraint impeding firms to operate at their optimal scale. If firms cannot operate at their optimal scale, this will affect the aggregate output and TFP. This hypothesis is consistent with observations by Kuznets (1966), Lucas (1978) and Tybout (2000), who argue that the average business size tends to increase with development.

In this paper we build a neoclassical growth model with adverse selection and endogenous firm size in order to examine and quantify the above mentioned relationships between the quality of property rights, the firm size distribution, per capita output and TFP. We find out that the introduction of adverse selection not only fills in a gap in the literature, it has also an important effect from the quantitative point of view.

Firms in our model are of two types, one (good firms) having higher productivity than the another (bad firms). Firms are of variable size, which means that firm size distribution is endogenous. Both types of firms finance their capital via a financial intermediary. However there are two types of imperfections in the financial market with affect financial contract between firms and financial intermediary: *i*) there is imperfect enforcement; *ii*) financial intermediaries cannot distinguish whether a firms is good or bad; that is, there is asymmetric information between financial intermediaries and firms, which involves an adverse selection problem. Imperfect enforcement in our model means that if firms default, the repayment of their debt can only be enforced up to a fraction of firms' cash flow. This fraction of firms' cash flow that can be seized by financial intermediaries is interpreted as an index of property rights quality. We find out that when property rights are perfect, or nearly perfect, there is neither borrowing constraint nor adverse selection problems, which means that the firm size distribution is optimal and the TFP attains the maximum possible level. When property rights are not perfect, firms are borrowing constrained due to two types of incentive constraints: *i*) non default incentive constraint, which means that financial intermediaries only lend to firms up to the point where the firm has incentive to repay her debt and not to default; *ii*) revelation constraint, which means that both types of firms have incentive to correctly revel their type. This last constraint is the one that it is specific to adverse selection problems. It turns out that this second type of constraint only affects the good type firms and that such constraint is tighter than the non default constraint. In this sense, good firms suffer tighter incentive constraint than bad firms. These incentive constraints entail borrowing constraints which generate two types of distortions: *i*) firms may be borrowing constrained, which implies that the firm size is smaller than their optimal size; *ii*) since the revelation constraint only affects good type firms, and this constraint is tighter than the non default constraint, it turns out that the amount of resources that are allocated to good type firms is smaller than the optimal level. This last distortion is specific to the adverse selection problem. Both types of incentive constraints become tighter when property rights worse off. This twists the firm size distribution, which affects aggregate variables such as the TFP and the per capita income.

We calibrate the parameters of the model to match aggregate and firm size data on the US economy and we compare steady states with different quality of property rights enforcement. Variations in the quality of property rights generate large variations in output per worker, though not so large variations in TFP. The firm size distribution is also affected, a lower quality of property rights being associated, in general, to a smaller average firm size. Furthermore, the share of factors, capital and labor, going to good type firms decreases dramatically when property rights deteriorate (from around 80% to around 20% in the worse case). The reallocation of capital and labor from good to bad firms is due to adverse selection and amplifies the effect of weak property rights on the economy. According to our results, countries could increase their output per worker, on average, by 20 % if they would completely eliminate the adverse selection problem (but not the enforcement problem). The gains would be even higher for the less developed countries.

To map these results against data, we use available statistics on average firm size (which in our model evolves monotonically with the quality of property rights) for a cross section of 30 developing and developed countries. The data shows a clear positive correlation between the firm size and the output per worker. This correlation is noticeable not only between the two country income groups (middle and high income), but also within these groups. Our model based on differences in property rights and adverse selection explains a significant part from the output gap between US and the countries in our sample. Besides the experiment with average firm size, we perform an additional test using a measure for the debt recovery rate compiled by Djankov et all (2008) on the basis of a case study for 88 countries. The debt recovery rate appears to be strongly correlated with output, and also with the average firm size. Moreover, it renders additional testimony that our model, based on the differences in property rights and the adverse selection mechanism, is able to explain a significant part of the variation in output across countries.

Nonetheless, the model displays a more modest performance in generating differences in TFP. Like most neoclassical literature on growth, we looked at the economy in the aggregate and we consider that all the production is realized by the same technology in a single sector. But this approach has a limitation: Rajan and Zingales (1998) have shown that financial frictions do not affect symmetrically every sector, instead different industries differs in their dependence on external finance. There are sectors that have more financial needs due to their large scale of operation and that, consequently, are more vulnerable to financial marker imperfections than other sectors where the scale of operation are not that large. This feature has been enphasized in the paper by Erosa and Hidalgo (2008), Buera, Kabosky and Shin (2011) and Midrigan and Xu (2012). To incorporate this feature to the model, we propose an extension where, in addition to the two types of firms (representing the "large scale of operation" technology) there is a technology, called self-funding technology, which is not affected by the scale of operation and that can be directly financed by household. Consequently this type of technology do not need financial intermediaries and is not affected by the quality of property rights. A decrease in the quality of property rights determines a reallocation of resources from the large scale technology to the self-funding one. After these modifications, the model generate important differences in TFP that replicate remarkably well the across countries differences in TFP.

Our paper is related to the literature that studies how financial intermediaries emerge to improve resource allocation, starting with Townsend (1978) and continuing with Bencivenga and Smith (1991), Boyd and Prescott (1986), Greenwood and Jovanovic (1990) and a survey by Levine (1997). However, these papers do not focus on the firm size distribution and do not quantify the importance of adverse selection. The occupational choice and the endogenous firm size are elements employed also by Erosa (2001), but his main concern is the intermediation cost and not the quality of property rights. The theoretical contribution closest to our model is Erosa and Hidalgo (2008) who develop a theory of capital markets affected by imperfect property rights and asymmetric information. However, firm size, which is central to our results, is exogenous in their model and they focus on how low enforcement affects the allocation of resources among entrepreneurs of different types. Another important difference is that their contract is written before entrepreneur learns their type. In contrast with the adverse selection literature and our model, this assumption implies that a once that the contract is offered by a financial entrepreneur, a competing financial intermediary cannot offer a better contract to a particular type of entrepreneur and, consequently, cross-subsidy across types occurs at equilibrium.

There are also several quantitatively oriented studies, although fewer than the theoretical contributions. Amaral and Quintin (2010) propose a model with imperfect property rights, and provide calibrated simulations that reveal significant effects on output. Buera, Kaboski and Shin (2011) also measure the role of imperfect property rights for output and TFP. They provide a rich framework emphasizing the role of sector specific fixed costs that cause differences in scale of production and forward looking savings behavior to overcome financial constraints. Both Amaral and Quintin (2010) and Buera, Kaboski and Shin (2011), with their methodology and findings, are an important advance in the quantitative study of the aggregate implications of differences in property rights and financial development. However, they abstract from the adverse selection effect, which plays an important role in our analysis: we show that, once taken into account, adverse selection significantly increases the effect of property rights enforcement on output. We can also mention Greenwood et all (2013) who examine the effects of financial development on economic development with a model of costly state verification. However, their framework is not one of different quality of property rights, but one of different efficiency of the financial sector.

The paper is organized as follows. Section 2 presents the model economy and Section 3 the agents decisions. The optimal financial contract is discussed in Section 4, and the equilibrium and steady state are defined in Section 5. In Section 6 we study the implications of imperfect property rights and adverse selection for economic development. In Section 7 we propose an extension of the model with a self-funding technology. Section 8 concludes. Proofs and calibration details are collected in the Appendix.

2. The Model

This is an infinite horizon economy where the time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. There are three types of agents: households, firms and financial intermediaries.

2.1. Technology

There is one final good that may be used for consumption or for investment and is produced using three factors: capital, labor and managerial time according with the following production function at firm level:

$$y_{jt} = \begin{cases} \theta_j k_{jt}^{\alpha} l_{w,jt}^{\beta} & \text{if } l_{m,jt} \geq \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

where y_j denotes the production by the firm j, k_j is the capital used by the firm j, $l_{w,j}$ is the amount of labor, $l_{m,j}$ the amount of managerial time. θ_j, with $j \in \{g, b\}$, denotes a firm specific technological shock that may take two values: good θ_g or bad θ_b, where $\theta_g > \theta_b$. The technological shock allows us to model the asymmetric information in the capital market and also to generate a firm size distribution[2]. The firms receiving the good technological shock θ_g represent the more productive firms in the economy and will be referred to in this paper as "good type firms". The firms receiving the bad technological shock θ_b represent the less productive firms in the economy and will be referred to in this paper as "bad type firms". The probability of getting the good shock is $\nu \in (0, 1)$. For the maximization

[2]We used only two values, good and bad, for the technological shock for tractability (the adverse selection problem becomes difficult to solve for more types).

problem of the firm to be well defined, we assume that $\alpha + \beta \in (0, 1)$. This production function allows us to obtain an endogenous firm size. This type of production function has been used, among others, by Perera-Tallo (2003, 2011), Guner, Ventura and Xu (2007), Antunes and Cavalcanti (2007) and Amaral and Quintin (2010). Firms are perfectly competitive and there is free entry. Hence, their profits will be zero.

Firms finance the capital from a financial intermediary. This simplifying assumption helps the tractability of the model and we discuss it in section 6.3. Capital in the economy accumulates according to the standard neoclassical capital accumulation equation:

$$K_{t+1} = I_t + (1 - \delta)K_t \tag{2.2}$$

where K denotes the stock of capital, I investment and δ is the depreciation rate.

2.2. Households

There are many identical households, each with a continuum of members of measure one. Each household values streams of the final consumption good according to the utility function:

$$\sum_{t=0}^{\infty} (\frac{1}{1+\rho})^t u(c_t)$$

where $\rho \in (0, 1)$ is the discount rate of the household and c_t is the average consumption of the household's members. Each household member is endowed with one unit of labor every period, and may chose between becoming manager or worker. The assumption that households are composed of a continuum of agents implies that agents may diversify perfectly the risk associated with the technological shock θ_j.

2.3. Financial Intermediaries

We will assume that households cannot invest directly in firms (they do not have the ability to deal with the capital market imperfection examined in this paper). It will be the role of the financial intermediaries to borrow from households and lend to firms. We will also assume that there is free entry and perfect competition in the financial intermediation sector.

There are two types of imperfections in the capital market: First of all, financial intermediaries cannot observe the idiosyncratic technological shock of the firms (there is an asymmetric information problem). Second, in case the firm defaults, the financial intermediary can only enforce the contract with probability $\lambda \in [0, 1]$, which is an index of the quality of the property rights (imperfect property rights quality).

3. Agents Decisions

In this section we will examine the economic decisions taken by households and firms. The relationship between firms and financial intermediaries will be studied in section 4.

3.1. Household decision

At the beginning of every period t each member of the household makes an occupational choice between being a worker or a manager. The difference between being worker or manager is that the payment of the worker is independent of the idiosyncratic shock of the firm, while the payment of the manager is contingent on it. More precisely, if a member of the household decides to be a worker, she will earn a wage w_t. If she decides to be a manager, she will earn a payment w_{jt}^M contingent on the shock of the firm. That is, she will earn w_{gt}^M if the firm she manages receives a good technological shock ($\theta_j = \theta_g$, with probability ν) and w_{bt}^M if the firm receives a bad technological shock ($\theta_j = \theta_b$, with probability $1-\nu$). Hence the expected managerial wage will be $w_t^M = \nu w_{gt}^M + (1-\nu) w_{bt}^M$.

The household chooses at every period t the per capita consumption c_t, and the portion of household members who become workers denoted l_t, where $l_t \in (0, 1)$. The household's per capita assets at each period a_t are deposited with financial intermediaries which pay an interest r_t on them. The household takes as given the prices $\{r_t, w_t, w_t^M\}_{t=0,\infty}$ when making its decision on $\{c_t, l_{t,}\}_{t=0,\infty}$.

The household problem is:

$$\begin{aligned} &\underset{c_t, l_t}{Max} \sum_{t=0}^{\infty} (\tfrac{1}{1+\rho})^t u(c_t) \\ &s.t. \; c_t + a_{t+1} - a_t \leq w_t l_t + w_t^M (1 - l_t) + r_t a_t, \forall t \end{aligned} \tag{3.1}$$

That is, the household maximizes utility subject to the budget constraint. The left hand part of the budget constraint is the expenditure, that may be in consumption

c_t or in assets $a_{t+1} - a_t$ (the savings). This expenditures should be equal or smaller than income, the left hand side of the budget constraint. The income consist of labor income of household members that become workers $w_t l_t$, the labor income of household members that become managers $w_t^M(1 - l_t)$, and income from assets $r_t a_t$ (capital income). Note that the assumption that each household is composed by an infinite number of members, implies that households can perfectly diversify the risk of the manager payment. This is the reason why the "wage" associated to the labor devoted to management is the expected payment of managers $w_t^M = \nu w_{gt}^M + (1 - \nu) w_{bt}^M$.

The first order conditions of the household problem are:

$$u'(c_{t+1}) \frac{1 + r_{t+1}}{1 + \rho} = u'(c_t) \tag{3.2}$$

$$w_t = w_t^M \tag{3.3}$$

where the first equation is the Euler Equation, while the second is an arbitrage condition: the wage of the workers should be equal to the expected payment to managers. Since households can perfectly diversify the risk of the manager payment, the wage is equal to the expected payment of managers (otherwise, in equilibrium managers would receive some risk premium).

The following transversality condition should hold:

$$\lim_{t \to \infty} \frac{a_t}{\prod_{i=0}^{t} (1 + r_i)} = 0$$

3.2. Firms decision

At the beginning of period t, firms hire the managers, before the realization of the productivity shock. This means that the number of managers cannot be contingent on the realization of the technological shock. Since the productivity shock is at this stage not known, the firm will specify the payment to the manager contingent on the productivity shock. Then the productivity shock θ_j is realized. Once they realize their productivity, firms engage capital k_{jt}, which is financed through a financial intermediary, according to a contract we will analyze in the next section. After borrowing the capital, firms hire labor n_{jt}, and use it together with the capital to produce the unique final good. After they produce, they sell the output to the households and pay the workers the wage w for their labor services. Thus, they chose the amount of workers in order to maximize the difference between revenues (production) and labor costs; such difference will be referred to as cash flow:

$$\pi_j(k,w) = \max_{l_{w,j}} \theta_j k^\alpha l_{w,j}^\beta - w l_{w,j} \tag{3.4}$$

The first order condition of the above function says that firm chooses an amount of workers such that the marginal productivity of these is equal to the wage:

$$\beta\theta_j k^\alpha l_{w,j}^{\beta-1} = w$$

Substituting the above first order condition in the definition of cash flow, we obtain the following:

$$\pi_j(k,w) = (1-\beta)\left(\left(\frac{\beta}{w}\right)^\beta \theta_j k^\alpha\right)^{\frac{1}{1-\beta}}$$

As mentioned in subsection 2.3, the firms either fulfill the contract paying the principal plus the interest rate $(1+r_j)k_{jt}$ or they default. By defaulting, we mean that firms refuse to return the loan. When a firm defaults, the financial intermediary can recover the capital of the firm and with probability λ can take also the cash flow of the firm $\pi_j(k_j, w)$.

After the payment to the financial intermediary, managers are paid. It follows from the firm zero profit condition that managers receive the cash flow of the firms minus the payment to the financial intermediary.

4. The optimal financial contract

In this section we analyze the optimal contract between financial intermediaries and firms. Along this section we will consider both wages and deposit interest rates as given. Taking wages as given, we will denote the cash flow function simply as $\pi_j(k_j)$ (instead of $\pi_j(k_j, w)$), making the notation more compact. To simplify the exposition, along this section we will refer to the profits of firm before paying the managers simply as profits: $\pi_j(k_j) - (r_j+\delta)k_j$.

The financial intermediaries will offer, for each of the two types of firms (good and bad), a contract consisting of an amount of capital k_j and an interest rate r_j. Given that financial intermediaries act perfectly competitive, expected (average) profits of financial intermediaries should be zero. In order to define the expected profit of financial intermediary, we will use the indicator function $\chi_j(r_j, k_j; \lambda)$ to denote the range of interest rate and capital at which the firm of type j will not default:

$$\chi_j(r_j, k_j; \lambda) = \begin{cases} 1 & \text{if } \pi_j(k_j) - (1 + r_j)k_j \geq (1 - \lambda)\pi_j(k_j) \\ 0 & \text{if } \pi_j(k_j) - (1 + r_j)k_j < (1 - \lambda)\pi_j(k_j) \end{cases}$$

The above indicator function means that if $\chi_j = 1$, the firm fulfills the contract, while if $\chi_j = 0$, the firm defaults. Thus, the expected profit by financial intermediaries is as follows:

$$\begin{aligned} &\nu\left[\chi_g(1+r_g)k_g + (1\text{-}\chi_g)\lambda\pi_g(k_g)\text{-}(1+r)k_g\right] + \\ &(1\text{-}\nu)\left[\chi_b(1+r_b)k_b + (1\text{-}\chi_b)\lambda\pi_b(k_b)\text{-}(1+r)k_b\right] = 0 \end{aligned} \tag{4.1}$$

The revenue of a financial intermediary from a contract is equal to $(1+r_j)k_j$ when the firm fulfills the contract $(\chi_j = 1)$ and $\lambda\pi_j(k_j)$ when the firm does not fulfill the contract $(\chi_j = 0)$, while the cost of a contract in terms of the payment to depositors is $(1+r)k_j$.This condition will be referred to as financial intermediaries zero profit condition.

At equilibrium it should not be possible for a financial intermediary to make a better offer to any type j firm, without incurring a loss. Due to the two imperfections in the functioning of the financial markets (imperfect property rights quality and asymmetric information), the menu of contracts will maximize profits for each firm type, taking into account two incentive constraints: one of them in order that firms have incentives to repay their loans and another one in order that firms truly reveal their type.

The first constraint refers to the incentive for firms to repay the financial intermediary and it will be called the non-default constraint:

$$\pi_j(k_j) - (r_j + \delta)k_j \geq (1 - \lambda)\pi_j(k_j), j \in \{g, b\} \tag{4.2}$$

If a firm defaults, the financial intermediary gets the capital after depreciation $(1 - \delta)k$, and with probability λ she also gets firm's cash flow, while the firm gets the expected amount $(1 - \lambda)\pi_j(k_j)$. If the firm fulfills the contract she gets her cash flow minus the payment to the financial intermediary: $\pi_j(k_j) - (r_j + \delta)k_j$. This incentive constraint is saying that the firm is better off paying to the financial intermediary, in which case she gets $\pi_j(k_j) - (r_j + \delta)k_j$ than defaulting, in which case she gets $(1 - \lambda)\pi_j(k_j)$.

The second constraint refers to the incentive for each firm to correctly reveal her type, and we will call it revelation constraint:

$$\begin{array}{rl} \pi_b(k_b) - (r_b+\delta)k_b \geq & \max\{(1-\lambda)\pi_b(k_g), \pi_b(k_g) - (r_g+\delta)k_g\} \\ \pi_g(k_g) - (r_g+\delta)k_g \geq & \max\{(1-\lambda)\pi_g(k_b), \pi_g(k_b) - (r_b+\delta)k_b\} \end{array} \tag{4.3}$$

This incentive constraint says that the bad type firm should be better off to sign the contract destined to her own type, getting $\pi_b(k_b)-(r+\delta)k_b$ as a payoff, rather than pretending to be an good type, getting the payoff

$$\max\{(1-\lambda)\pi_b(k_g), \pi_b(k_g) - (r_g+\delta)k_g\}$$

The same should be true for the good type firm.

4.1. The equilibrium menu of contracts

Definition 4.1. *A menu of contracts $\{(r_g, k_g), (r_b, k_b)\}$ satisfying the financial intermediary zero profit condition (4.1) and the revelation constraints (4.3) represents an equilibrium menu of contracts if*

1. $\forall j \in \{g, b\}$ there is no other contract (r'_j, k'_j) for the type j in which the firm j is better off $\pi_j(k'_j) - (1+r'_j)k'_j > \pi_j(k_j) - (1+r_j)k_j$ and in which financial intermediary gets at least zero profit $\chi_j(1+r_j)k_j + (1\text{-}\chi_j)\lambda\pi_j(k_j) \geq (1+r)k_j$ and in which the revelation constraints (4.3) hold.

2. There is no other menu of contracts $\{(r'_g, k'_g), (r'_b, k'_b)\}$, in which financial intermediary gets at least zero profit (4.1) and in which $\pi_j(k'_j) - (r'_j+1)k'_j \geq \pi_j(k_j) - (1+r_j)k_j \ \forall j \in \{g, b\}$, being the last inequality strict for one of the types, and in which the revelation constraints (4.3) hold.

Thus, a menu of contracts is an equilibrium if it satisfies the financial intermediary zero profit condition (4.1) and the revelation constraints (4.3), and it is not possible to find, for one or both firm types, a better contract where non-negative financial intermediary profit condition is satisfied. It is still possible a different contract in which both types have the same contract and in which the bad type defaults: it is the pooling contract. Nevertheless, it is well known that this type of pooling contract is never an equilibrium. Furthermore, Chapter 1 shows that if there are enforcement cost (or state verification cost) in case of default, and these costs are high enough, the pooling contract is never superior to the separating menu of contract. Thus, we do not consider pooling contracts.

We note that an equilibrium menu of contracts will have $r_b = r_g = r$. Otherwise, the financial intermediary would be making positive profits with the contract of at least one firm type. This couldn't be sustained as an equilibrium because it would always be possible to offer a better contract to this type.

As the good type firm has higher productivity, and the interest rate of both types are equal, non default constraint (4.2) implies that good type firm gets a larger amount of capital than the bad type firm[3]. This means that the good type firm has never the incentive to pretend to be a bad type. Only the bad type firm has incentives not to correctly reveal her type. This means that the revelation constraint (4.3) is never binding for the good type firm and consequently is irrelevant for the subsequent analysis. From the perfect competition in the financial sector it follows that the bad type firm is offered the best contract such that the financial intermediary gets zero profit and the Non-Default Constraint is satisfied for the bad type firm. This means that if the bad type firm is asking for the contract of the good type, it is because she plans to default. If she wouldn't plan to default, she would be better off accepting the contract offered to her type. Thus, the revelation constrain for the bad type is the only revelation constraint which is relevant for the equilibrium and may be rewritten as follows:

$$\pi_b(k_b) - (r_b + \delta)k_b \geq (1 - \lambda)\pi_b(k_g) \tag{4.4}$$

It is possible to prove that for the good type the revelation constraint (4.4) is always tighter than its non-default constraint [4]. Hence, the only incentive constraint relevant for the good type is the revelation constraint.

To sum up, if $\{(r_g, k_g), (r_b, k_b)\}$ is a menu of contracts, then $r_g = r_b = r$ and:

$$\begin{aligned} k_b = \quad & \arg\max_k \pi_b(k) - (r + \delta)k \\ & \quad s.t.\ \pi_b(k) - (r + \delta)k \geq (1 - \lambda)\pi_b(k) \end{aligned} \tag{4.5}$$

$$\begin{aligned} k_g = \quad & \arg\max_k \pi_g(k) - (r + \delta)k \\ & \quad s.t.\ \pi_b(k_b) - (r + \delta)k_b \geq (1 - \lambda)\pi_b(k_g) \end{aligned} \tag{4.6}$$

Thus, the menu of contracts is characterized by the following features: i) the interest rate that financial intermediaries charge firms is the depositors interest rate; ii) there is no default in equilibrium since non default constraint is satisfied,

[3] Chapter 1 offers a formal proof for the statements in this section.
[4] Chapter 1 offers a formal proof for the statements in this section.

this and the above features guarantee the zero profits of financial intermediaries; *iii*) The only incentive constraint that is relevant for the bad type is the non default constraint, since the good type never has incentives to cloak his type. The only incentive constraint that the good type faces is the revelation constraint, which is always tighter than the non-default constraint for the good type. *iv*) The menu of contracts is such that firms of each type maximize profits subject to the relevant incentive constraint for each type (the non-default constraint for the bad type and the revelation constraint for the good type).

The non default constraint (4.2) may be rewritten as follows:

$$\lambda \frac{\pi_j\left(k_j\right)}{k_j} \geq (r+\delta), j \in \{g, b\} \tag{4.7}$$

The above equation means that the firm has incentive to fulfill the contract if the expected average payment to the financial intermediary in case of default $\lambda \pi_j\left(k_j\right)/k_j$ is higher than in case of fulfilling the contract

The upper part of Figure 1 displays the non default constraint (4.2) for the bad type: the non default constraint is satisfied when the profit in case of fulfilling the contract, which is hump-shaped, is superior to the profit in case of default, which is an increasing curve. In the lower part of Figure 1 is represented another version of the non default constraint, corresponding to equation (4.7): the bad type firm has incentive to fulfill the contract if the expected average payment to the financial intermediary in case of default, which is a decreasing function in k_j, is higher than in case of fulfilling the contract, which is constant. It is apparent from lower part of Figure 1 that the non-default constraint holds when the amount of capital is smaller than a certain threshold capital in which the non-default constraint holds with equality. Such level of capital is the maximum amount that the bad firm is going to be able to borrow. This will be called the non default borrowing limit, and we will denote it by k_b^{nd}:

$$k_b^{nd} \overset{def}{\Leftrightarrow} \pi_b\left(k_b^{nd}\right) - (r+\delta)k_b^{nd} = (1-\lambda)\pi_b\left(k_b^{nd}\right)$$

Figure 2 displays the revelation constraint which affects the good type: the profit of the bad firm fulfilling her contract should be larger than the profit of bad firm when she accepts the contract of the good type and defaults. It follows from the fact the cash flow function is strictly increasing (which implies that the profit of the bad firm in case of default is increasing) that the revelation constraint holds when the amount of capital is smaller that certain threshold capital in which the revelation constraint holds with equality. Such level of capital is the maximum amount that the good firm is going to be able to borrow. This will be called the revelation borrowing limit and we will denote it by k_g^{rv}:

$$k_g^{rv} \Leftrightarrow \pi_b(k_b) - (r+\delta)k_b = (1-\lambda)\pi_b\left(k_g^{rv}\right) \text{ (and } k_g^{rv} > 0)$$

Thus, another way to write the menu of contracts is as follows:

$$\begin{aligned} k_b = \quad & \arg\max_k \pi_b(k) - (r+\delta)k \\ & s.t.\ k \leq k_b^{nd} \end{aligned} \tag{4.8}$$

$$\begin{aligned} k_g = \quad & \arg\max_k \pi_g(k) - (r+\delta)k \\ & s.t.\ k \leq k_g^{rv} \end{aligned} \tag{4.9}$$

4.2. Financial contract and the quality of property rights

In this subsection we characterize the optimal financial contract that the financial intermediary offers in equilibrium to each firm type (good and bad), as a function of the quality of property rights λ.

When the quality of property rights goes down, the punishment for default falls as well, which implies that the maximum amount of capital that the bad firm can receive, the non default borrowing limit k_b^{nd}, decreases. Thus, after a drop in property rights enforcement, it is more likely that the firm is constraint, or, in order words, that the non-default constraint is binding. If the bad type firm was already constrained, then the fall in the property rights enforcement may further reduce the non default borrowing limit and consequently the amount borrowed by the bad type firm, diminishing her profits.

But the fall in property rights enforcement also affects the good firm. As we have seen, the fall in property rights enforcement reduces the profits of the bad firm (left hand side of the revelation constraint 4.4), which reduces the "positive incentives" to reveal her type. Furthermore, a drop in property rights enforcement also reduces the punishment in case the bad firm does not reveal her type (see right hand side of the revelation constraint 4.4). Thus, the fall in property rights enforcement reduces also the revelation borrowing limit k_g^{rv}, which affects the good firms, making more likely that such firms are constrained and reducing the capital received by them in case that these firms were already constrained.

Summarizing, the fall in property rights enforcement will reduce both the non default borrowing limit for bad firms and the revelation borrowing limit for good firms, making more likely that firms are constrained and reducing the amount of capital that good firms borrow in case that these firms are already constrained. Thus, a fall of property rights enforcement tights the incentive constraints and affects firm size distribution by making firms smaller than their optimal size.

The unconstrained maximization of profits implies optimum amounts of capital for each of the two firm types, which we will denote by k_j^* with $j \in \{g, b\}$:

$$k_j^* = \arg\max_k \pi_j(k) - (r+\delta)k \Leftrightarrow \frac{\partial \pi_j\left(k_j^*\right)}{\partial k} = (r+\delta)$$

It follows from the implicit function theorem that good firms receive more capital than bad ones (i.e. $k_g^* > k_b^*$). We will use this particular case a benchmark/starting point for our analysis.

Proposition 4.2. *There are two thresholds for the quality of property rights enforcement*[5] λ_1 *and* λ_2, *where* $1 > \lambda_2 > \lambda_1 > 0$ *and such that: g) If* $\lambda \geq \lambda_2$ *then neither the good or the bad firms are constrained:* $k_j = k_j^*$. *ii) If* $\lambda \in [\lambda_1, \lambda_2)$ *then good firms are constrained,* $k_g < k_g^*$, *while bad firms are not,* $k_b = k_b^*$, *the amount of capital received by good firms* k_g *increases with* λ. *iii) If* $\lambda < \lambda_1$, *both types of firms are constrained and the amount of capital received by each type is the same:* $k_g = k_b < k_b^*$; *furthermore, the amount of capital that firms receive increases with* λ *(proof in appendix).*

Thus, when the quality of property rights enforcement drops, the incentives for firms to fulfill the contracts and to truly reveal their type fall as well. This makes incentive constraints tighter, reducing the non default borrowing limit, which affects bad firms, and the revelation borrowing limit, which affects good firms. There are three possible situations depending on the quality of property rights. When the quality of property rights is good enough but not necessarily perfect, i.e. larger than λ_2, the incentive constraint are not binding and firms choose their optimal capital level. When the quality of property rights lies in a middle range, $\lambda \in [\lambda_1, \lambda_2)$, only the good firms are constrained while bad firms are not. In this sense, good firms suffer an incentive constraint (the revelation incentive constraint) that is tighter than the one that affects bad firms (the non default constraint). Finally, when the level of contract enforcement is poor, $\lambda < \lambda_1$, both firms are constrained and receive the same inefficiently small amount of capital.

[5] The exact definition of these thresholds are: $\lambda_1 \equiv \frac{\alpha}{1-\beta}$ and $\lambda_2 \equiv 1 - \left(\frac{\theta_b}{\theta_g}\right)^{\frac{\alpha}{(1-\beta)(1-\alpha-\beta)}} \left(\frac{1-\alpha-\beta}{1-\beta}\right)$

Furthermore, when firms are constrained and the level of contract enforcement improves, borrowing limits expand making firms closer to their optimal level and consequently more productive.

Thus, the link between property rights quality and productivity is clear from proposition 4.2. Weak property rights make firms have an inefficiently small size. Furthermore, weak property rights affect more good firms, and affect the way capital is distributed to firms, pouring relatively more resources in bad and less productive firms and less resources in good and more productive firms. These two mechanisms, the reduction of firms size and the redistribution of resources from more productive to less productive firms, imply that the quality of property rights will affect positively the productivity. This will be analyzed in more detail in section 6.

5. Equilibrium and Steady State

An equilibrium is an allocation $\{c_t, a_t, \{k_{jt}, r_{jt}, l_{w,jt}\}_{j\in\{g,b\}}, n_t, l_t\}_{t=0}^{\infty}$ and a vector of prices $\{r_t, w_t, w_{gt}^M, w_{bt}^M, w_t^M\}_{t=0}^{\infty}$ such that $\forall t \in \{0, 1, 2, ...\}$:

1. Households maximize their utility according with problem (3.1).

2. Financial intermediaries offer to each firm of type $j \in \{g, b\}$ a contract $\{k_{jt}, r_{jt}\}_{j\in\{g,b\}}$ that is an equilibrium contract according to definition 4.1.

3. Type j firm chooses how many workers $\{l_{w,jt}\}_{j\in\{g,b\}}$ to hire in order to maximize the profits, taking as given the worker wage w_t, and the amount of capital k_{jt} and the interest rate r_{jt} offered by the financial intermediary.

4. Zero profit condition (or free entry condition):
$w_{gt}^M \varepsilon = \pi_g(w_t, k_{gt}) - r_{gt}k_{gt}$
$w_{bt}^M \varepsilon = \pi_b(w_t, k_{bt}) - r_{bt}k_{bt}$
$w_t^M = \nu w_{gt}^M + (1-\nu) w_{bt}^M$.

5. Labor market clears:
$n_t[\nu l_{w,gt} + (1-\nu) l_{w,bt}] = l_t$
$n_t \varepsilon = 1 - l_t$.

6. Capital market clears:
$n_t[\nu k_{gt} + (1-\nu) k_{bt}] = a_t$.

where n_t denotes the per capita number of firms. Thus, households, firms and financial intermediaries should maximize their respective objective function in equilibrium. This means that households maximize their utility according with household maximization problem (3.1), the menu of contract in accordance of the maximization problems (4.5)-(4.6) (or (4.8)-(4.9)) and firms chose the amount of labor that maximize profits according with problem (3.4). Furthermore, free entry conditions hold, which means that profits by firms are zero. This implies that the payment to the managers in the good type firm $w_{gt}^{M}\varepsilon$ should be equal to the cash flow minus the payment to the financial intermediaries $\pi_g(w_t, k_{gt}) - r_{gt}k_{gt}$ and the same should be true for the bad type of firms ($w_{bt}^{M}\varepsilon = \pi_b(w_t, k_{bt}) - r_{bt}k_{bt}$). Finally, markets should clear, which means that in the labor market the per capita demand of workers by firms should be equal to the per capita supply of workers by households l_t. The per capita demand for workers is equal to the per capita number of firms n_t multiplied by the average (or expected) demand for workers by firms $[\nu l_{w,gt} + (1-\nu)l_{w,bt}]$. The same should occur for managers: the per capita demand for managers by firms, which is equal to the per capita amount of firms n_t multiplied by the demand for managers by each firm ε, should be equal to the supply of workers by households, $1 - l_t$. In the capital market, the per capita demand for capital, which is equal to the per capita number of firms n_t multiplied by the average demand for capital by firms $[\nu k_{gt} + (1-\nu)k_{bt}]$, should be equal to the supply of capital by households a_t.

Steady state is an equilibrium where $\{c_t, a_t, \{k_{jt}, r_{jt}, l_{w,jt}\}_{j\in\{g,b\}}, n_t, l_t\}_{t=0}^{\infty}$ and $\{r_t, w_t, w_{gt}^{M}, w_{bt}^{M}, w_t^{M}\}_{t=0}^{\infty}$ are constant over time.

6. Economies with different Quality of Property Rights

In this section we consider steady state equilibria of economies that differ in only one aspect: the quality of property rights enforcement λ. We analyze how the output, the Total Factor Productivity (TFP) and the average establishment size change when we move from a lower to a higher quality of property rights enforcement.

We are aware that our quantitative experiments over-estimate the importance of financial frictions due to several reasons. Due to the complexity of adverse selection problems, we abstract from the dynamic aspects of the firm. Thus, we do not incorporate to the model the existence of internal funds or accumulated capital to collateralize loans, we do not consider dynamic contracts or the possible correlation of shocks. All these features would reduce the incentive problems as Cole, Greenwood, and Sanchez (2012) have shown. Furthermore, we consider in our calibration that all the heterogeneity in establishment size in the data is driven by the shock process on productivity, without taking into account other important factors (like age).

It is important to notice that in our benchmark case, with perfect property rights, the technology presented in the model implies that the aggregate production function is of the Cobb-Douglas type:

$$y = (k)^{\alpha} (\Gamma \times l)^{1-\alpha}$$

where $\Gamma = \left[\left(\frac{1-\beta-\alpha}{\varepsilon}\right)^{1-\alpha-\beta} \frac{\beta^{\beta}}{(1-\alpha)^{1-\alpha}} \left[\nu \left(\theta_g\right)^{\frac{1}{1-\alpha-\beta}} + (1-\nu) \left(\theta_b\right)^{\frac{1}{1-\alpha-\beta}} \right]^{1-\alpha-\beta} \right]^{\frac{1}{1-\alpha}}$ and l is the per capita amount of labor devoted to production (both workers and managers). Following Hall and Jones (1999), we define the TFP of the economy such that $y = k^{\alpha}(TFP \times l)^{1-\alpha}$, where y, k and l are respectively per capita output, capital and labor of the economy. Throughout this paper we consider managers as part of the working force so the total labor is unity, which means that the TFP can be calculated as:

$$TFP = \left(\frac{y}{k^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{6.1a}$$

We note that in our model, as population is constant and equal to unity, the *aggregate* output and capital can also be considered as output and respectively capital *per capita*. This definition of TFP has the advantage that if we consider a neoclassical growth model with the Cobb-Douglas production function, there is a one to one relationship between the ratio of per capita GDP in two countries in the steady state and the ratio of TFP in these countries:

$$\frac{y^{country\ A,ss}}{y^{country\ B,ss}} = \frac{TFP^{country\ A}}{TFP^{country\ B}}$$

Thus, with this definition we will get a clearer idea of the difference between the predictions of our model and the predictions of the neoclassical growth model

The following proposition establishes in our framework a formal relationship between the quality of property rights enforcement and the aggregate productivity.

Proposition 6.1. *The aggregate productivity (TFP) of the economy at the steady state is increasing in the quality of property rights (proof in appendix).*

The intuition is as follows. When the quality of property rights enforcement is very low, the financial intermediaries cannot offer the optimal amount of capital to any of the two firm types. As the quality of property rights starts improving, both firm types benefit from more capital, getting closer to their optimal production level (a higher quality of property rights allows the financial intermediaries to punish harder firm misbehavior). Once the economy reaches a sufficiently high quality of property rights, the financial intermediaries can separate between the two firm types, which permits them to allocate a relatively higher share of capital to the highly productive firms, in accordance to their larger optimal scale of production. This amplifies the increase in TFP, as the financial system becomes better at fulfilling its function of allocating productive resources in the economy. In this way the financing of production is able to overcome the imperfections of the credit market which act in our model as an impediment to development.

6.1. Calibration

We chose the parameter values so that our model, with perfect quality of property rights (λ equal to one), matches at the steady state data for the United States economy. We take the United States economy as a good approximation, for our purposes, of a distortion-free economy (having no significant problems with the quality of property rights). Table 6.1 offers a summary of the calibration procedure.

Parameter	Value	Target
α	0.36	Capital elasticity based on Cooley and Prescott (1995)
δ	0.09	Ratio between investment and capital
ρ	0.05	Capital-output ratio
ν	0.16	High productive firms are the ones who have more employees than the average number of employees (source 2002 US Economic Census)
β	0.58	The share of labor and the share of output corresponding to the high productive firms (source 2002 US Economic Census)
ε	3.9	Average number of employees for manufacturing establishments (source 2002 US Economic Census)
$\frac{\theta_g}{\theta_b}$	1.24	The share of output corresponding to the high productive firms

Table 6.1: Parameter values (source: authors calculations)

Some parameters like α, δ, r are widely used in macroeconomic models and we follow the standard procedure used in the literature which consists in choosing the parameters that make the model match some statistical regularity of the US economy. Other parameters relate to the firm size distribution and their calibration is based on the replication of relevant features for the firm size distribution in the US.

The calibration of the macroeconomic parameters is relatively clear-cut. Thus, for α, the share of capital in total output, we apply the methodology described in Cooley and Prescott (1995) on data from the National Income and Product Accounts. The share of capital α averages 0.36 for the period 1950-2010. We compute, using the same data source, the depreciation rate δ as the ratio between investment and capital and we obtain a value of 0.09.

At steady state the discount rate is equal to the net interest rate. We compute the net interest rate using the formula:

$$r = \alpha \frac{Y}{K} - \delta \tag{6.2}$$

For the capital-output ratio, we use data from the Bureau of Economic Analysis for the period 1950-2010. We define the stock of capital as being comprised by private fixed capital (nonresidential plus residential), durable goods and inventories. The definition for output Y is the same we use when computing the capital share α. The result is a capital-output ratio of 2.42 and a net interest rate of 0.05.

We are left with four parameters: the coefficient of labor in the production function β, the managerial time ε, the proportion of highly productive establishments ν and the ratio between the levels of high type and low type productivity $\frac{\theta_g}{\theta_b}$. For these parameters, we try to match in the best possible way the distribution of firm size generated by our model with the distribution of firm size given by available data for the U.S. We use the 2007 U.S. Economic Census which provides information on number of employees, valued added and other variables for the establishments grouped by the North American Industry Classification System (NAICS)[6]. Appendix 9.3 offers supplementary details on the firm size calibration.

First we need to decide which industry sectors from NAICS we want our model to describe. Our objective is to make cross-country comparisons regarding the importance of different property rights enforcement on output, firm size and TFP, therefore we would like to use data on firm size as comparable as possible across countries. Various industry sectors covered by NAICS vary as importance across economies; given the different optimal firm size across industry sectors, we focus

[6]As a classification of economic activities used in North America, NAICS is based on the United Nations' Standard Industrial Classification (SIC), just as the Statistical Classification of Economic Activities in the European Community (NACE).

The data from the 2002 U.S. Economic Census that we used is freely available at the U.S. Census Bureau website.

on manufacturing. In this way, we try to minimize the effect of different industry structure across countries (which is not the focus in our model) and to "isolate" as much as possible the effect of property rights from other factors that may affect firm size.

In our model there are two types of firms, larger (and more productive) and smaller (and less productive). The US Economic Census groups establishments in 10 size classes (starting with '1-4' employees up to 'more than 2500' employees, see Table 9.1). We note that the average value added per employee is increasing with firm size. Hence, the Census data seems consistent with the model's implication that the larger firms are also the more productive ones.

We need to match the firm size classes reported in the Census to the two firm types in our model. For this purpose we take those size classes employing on average more workers than the sector average to belong the "good" type (with larger and more productive firms). In this way we avoid imposing a definition of what is a 'large' firm. From Table 9.1 is becomes apparent that in our setting firms will be 'large' (or belong to the 'good' type) if they employ 50 or more workers.

The resulting share of large firms is 16% of the total number of firms, and we use it to match the share of labor and output corresponding to large firms. In this way we obtain the coefficient of labor in the production function β equal to 0.58. The managerial time per establishment, ε, is determined by the average establishment size in the data of 55,3 employees. Finally, the ratio between the levels of high type and low type firm productivity $\frac{\theta_g}{\theta_b}$ of 1.24 is obtained from matching the share of output corresponding to large firms. The resulting returns to scale $\alpha + \beta$ are 0.94, close to values used by Guner, Ventura and Xu (2006), Basu and Fernald (1997), Chang (1998) or Antunes and Cavalcanti (2007).

6.2. Results

Our experiments consist of comparing steady state equilibria in economies that differ in their quality of property rights, indexed by the parameter λ. In these comparisons we focus on output, average establishment size and total factor productivity.

The output is plotted in Figure 3 against the quality of property rights λ (in continuous line). It can be noticed that our model is consistent *qualitatively* with the empirical positive correlation between property rights quality and output. As property rights improve, relatively more capital and labor are hired by the high productivity firms: in Figure 4 it can be seen that the share of labor going to good firms is increasing from 0.2 when property rights quality is close to zero up to 0.8 when enforcement is perfect.

The model is able to generate large output differences between countries, and it could even account for "development disasters" (assuming the parameter λ reflecting the quality of property rights is low enough). Although this parameter is difficult to measure, in the next section we will use some subsidiary implications on average firm size (which is observable) to compare the predictions of the model with the data.

Figure 5 depicts the behavior of firm size, for good firms, bad firms and the economy average, as property rights quality varies. Here there are two effects at work. The first is the optimal financial contract effect[7]: taking the wage and interest rate as given, the two firm types will receive more capital as property rights improve. Once the bad type firm has reached its optimal scale of production, only the good type continues to receive more capital, until reaching its own optimal scale. The second effect is due to general equilibrium interactions. The improvement in property rights increases the wage, but not the steady state interest rate; this implies that firms use a lower ratio labor/capital, which acts towards reducing the labor employed by firms and therefore their size. This second type of effect can be observed in the firm size of bad firms, which reach maximum size when property rights are at the threshold level λ_2, at which bad firms cease to be constrained (see proposition 4.2). Thus, in a certain middle interval (between λ_2 and λ_1) bad firms are oversized when they are compared with the size of bad firms in the benchmark case of perfect property rights. For the good firms, the equilibrium contract effect outweights the wage effect, resulting in a firm size increasing with enforcement. Having these opposed effects at work, the resulting average firm size will depend on the quantitative importance of each of them. For the numerical values employed in our model, we get an increasing average firm size up to the point when bad type firms reach their maximum scale ($\lambda = \lambda_2$). Once the bad firms have reached their maximum size, general equilibrium effects will make the average firm size to slightly decrease before going up to its benchmark level of 41.9 employees for the case of perfect enforcement. The result is an average firm size which is monotonic for the most part of the interval considered.

Concerning aggregate productivity, we have shown in Proposition 6.1 that in our model higher property rights enforcement implies higher productivity. Now we are able to look at the quantitative importance of this effect. Figure 6 plots the TFP against the property rights parameter (in continuos line). The model

[7] We could also name this effect as the "pure" property rights effect.

is only able to generate variation across countries in TFP of up to 30% due to differences in enforcement. Obviously, these variation in TFP are too narrow to be able to explain the variation obtained in the data (see for instant Hall and Jones, 1999). To understand the mechanism behind, notice that the fixed cost consist of a labor cost, which implies that the labor intensity of the firm declines with the firm size. Thus, when property rights worsen, firms size shrinks, reducing the capital intensity of the firm, which implies that at the steady state the per capita capital declines more the per capita income; this entails that TFP does not declines much (see definition of TFP in equation 6.1a). Consequently, the effect of reducing the per capita capital seems to be the most important from the quantitative point of view.

6.3. The importance of adverse selection

A main novelty of our quantitative study has been to explicitly take into account the adverse selection in the neoclassical framework. At this stage we are able to verify if adverse selection has a significant influence on output and productivity, by comparing the results of the model with and without adverse selection. The predictions of the model are illustrated in Figure 3 for output and in Figure 6 for productivity (TFP): the continuos lines correspond to the economy with adverse selection, while the dotted lines represent the economy without adverse selection.

It is apparent from these figures that with adverse selection, the model predicts a stronger effect on output and productivity. To make this point clearer, we computed the output gain for a country that would eliminate the adverse selection friction (see Figure 7). By this feat, most countries would achieve a one-off increase in output per worker of around 20%, clearly a non negligible quantity[8]. One could argue that we assumed that all capital used in production is intermediated, while in reality, not all capital is intermediated; hence the effects of property rights and adverse selection are overstated in our model. Amaral and Quintin (2010) measured the stock of intermediated capital as the sum of the credit firms obtain from banks and from issuing bonds, and the funds they obtain from issuing stock. In this sense, all funds that are brought by the manager can be considered intermediated capital[9]. They find that in the US most capital is intermediated,

[8] To illustrate the importance of such an increase in output: with the average growth rate of output per worker in the US for the last 50 years, of approximately 1,5%, a country would need more than a decade to increase its output by 20%.

[9] They underline that retained earnings are funds that corporations effectively borrow from their shareholders, hence in this framework can be treated as external, instead of internal finance.

which can be explained by the fact that corporations are the leading form of ownership, and most of their capital is financed either by debt or stock. These findings suggests that, while not all capital is intermediated (especially in less developed economies), most capital used in production in a certain country is subject, one way or the other, to the quality of property rights enforcement in that country.

Why does adverse selection display such a strong effect on output? Our analysis suggests that this informational friction affects mainly the highly productive firms in an economy. If we take out adverse selection from our model, highly productive firms receive a higher amount of capital in equilibrium, as the low productive types are not allowed to deviate and choose the contract of the high type. On the contrary, with adverse selection, in equilibrium financial intermediaries cannot offer to the highly productive firms a sufficient amount of capital because low productive firms can falsely report their type.

Therefore, in our theory, the quality of property rights affects both the low and the highly productive firms, while adverse selection tends to punish harder the highly productive firms. One implication for development is that policies aimed at reducing informational frictions may be worth pursuing (in terms of efficiencies to be gained at the aggregate level). Amaral and Quintin (2010) and Buera, Kaboski and Shin (2011) performed a quantitative analysis of how imperfect enforcement affects development, and found those effects to be significant. However they did not specifically include an informational imperfection in their model. Erosa and Hidalgo (2008) performed a study with asymmetric information, but their focus was not quantitative. Our model provides an answer on how the quantitative findings about the relationship between enforcement, output and productivity are affected by the introduction of informational frictions. We view our paper as complementary to these contributions, as it approaches quantitatively the relationship between enforcement and development, considering also the effect of adverse selection.

6.4. Model vs cross-country data

In this section we compare the predictions of the model regarding output per worker with data available for several countries. To compute the predicted output per worker, we feed in data for the average firm size. In our model average firm size varies monotonically with the quality of property rights[10]. One could ask why we did not directly used data on the quality of property rights. Qualitative ratings of the law enforcement (e.g. those done by the International Country Risk Guide), while certainly useful for a relative ranking of countries (such as those performed by LaPorta et all 1996), do not offer absolute values needed to feed the model. Djankov et all (2008) use a case study to construct a measure for the efficiency of debt enforcement for 88 countries. While providing a quantitative measure of property rights quality, there are limits in extending the results of their case study to the overall economy. Nonetheless, at the end of this section we will use their quantitative measure as an additional test for our model.

We investigated the publicly available data on firm size, considering both the quality and comparability of data for our purposes. Countries that undertake periodically an economic census may be expected to have more reliable data on firm size. Data from surveys may be sometimes less prone to cross country comparisons if the surveys were not carried out with a similar methodology[11]. Taking account of this, we used as a main data source the OECD database on Structural Business Statistics, which contains the number of employees and the number of firms for all businesses and by industry sector for the OECD countries. The advantage of this source is that data are organized based on a common industry classification, ISIC rev 3. We added the data for Argentina from the economic census, whose national classification is compatible with ISIC rev 3 used by OECD. In this way we have a sample of 30 countries, consisting of countries usually characterized as either 'middle income' or 'developed'. We could not include any 'low income' countries in our sample. We could not find any reliable data (e.g. economic census) which is publicly available. There are some surveys but the coverage seemed insufficient and the average firm size is highly dependent on the coverage[12].

[10] As seen in previous section, there is a slight variation in monotonicity of average firm size when the quality of property rights is very close to one, but this does not restrict our analysis. From a conceptual point of view, countries in that interval have a level of development very similar to US, and are not the object of our study. From a practical point of view, it does not affect the sample we use.

[11] Different purpose of the survey, different geographical or size class coverage may significantly influence the results of the survey. This can be seen for instance in data available for the same country (India, China) from more sources (Asian Development Bank or United Nations' Statistics Division).

[12] In general, data on firm size for lower income countries is hardly available and of much lower quality than the data from developped countries. The high level of non-declared activities, usually combined with an insufficient statistical capacity imply that it is difficult for these countries to compile data with similar methodology (and reliability) as in the developped countries.

The average firm size varies significantly across the one digit level ISIC sectors (mining, manufacturing, construction, transport, etc.) - within and across the countries in our sample. Given that our model abstracts from relative shares of sectors in total economy, and factor shares vary across sectors (Herrendorf and Valentinyi 2008), we decide to focus on manufacturing in our cross country comparisons, which is also consistent with the calibration procedure[13]. The data on two digit level manufacturing industries reveals, across countries, that there is still a significant variation in firm size also within the manufacturing sector. For example, for the U.S., average firm size varies from 19 for machinery and equipment n.e.c. to 78 for energy, chemical and plastic products. However, this issue is not the object of our analysis. Therefore, for each country in our sample, we compute the average firm size as a weighted average of firm size in the two digit industries, keeping the industry weights constant and equal to the ones of US, which is the benchmark country in our analysis.

As a measure of output, from the Penn World Table (PWT) version 7.0 we take the GDP per worker (rgdpwok). The variable, used also by Amaral and Quintin (2010), is intended to control for differences in rates of employment and price levels across countries (which are not the focus of our analysis). Although for the countries in our sample the GDP per worker relative to US displays a certain variation over time, we are interested in its long term average. We used the average for the period 1995 up to most recent data, for two reasons: on the one hand, for this period the series are more stable and at the same time, the closer we are to present, generally the more reliable can be considered the series.

Figure 8 illustrates the relative output predicted by the model given the country data on average firm size, and the relative output implied by PWT data. The triangles are from model simulations, and the rhombuses represent country-level data. For both the model and the data, the quantities are normalized by the US level. First of all, the country data confirm the prediction of the model. There is a clear positive correlation between the average firm size and the relative GDP per worker, as shown by the regression line from country data. We find that the model's predictions for output are reasonable given available data. If we compare the data regression line with the model prediction, we note that in general the model predicts a higher GDP per worker than in the data. The unexplained

[13]The distribution of firm size is quite dispersed. Although it could be rightfully argued that the mean does not fully describe the distribution, for the purpose of our cross country comparisons the mean seems to works well as a summary indicator of the firm size distribution and it has been used in several studies like Guner et all (2008) or Amaral and Quintin (2010).

part could be due to the fact that we abstracted from other relevant factors for development (such as human capital differences). Nonetheless, the differences in the quality of property rights combined with the effects of adverse selection seem able to explain a great deal of the output disparities between U.S. and the rest of countries in our sample, especially considering that we are varying one single factor - the quality of property rights - across countries.

The data covers both countries that could be characterized as having a 'high income' and countries with a 'middle income'. While we have seen that the quality of property rights plays an important role for sample of countries as a whole, it may be useful to have a look also separately at each income group. Although there is no universal, agreed-upon definition for what makes a country developing versus developed, international organizations (IMF, UN, the World Bank) classify countries based on various criteria such as GNI per capita, or Human Development Index. In our case we use the variable at hand, the GDP per worker. Figure 9 displays how the data and the model behave for the countries in our sample with a GDP per worker of less than 60% of the US level, compared to the rest[14]. There are two empirical observations that deserve to be highlighted. First of all, the positive correlation between the property rights quality and firm size, on one hand, and relative GDP per worker, on the other hand, is not driven only by having in our sample two different income country groups, but exists also within these groups, as it is apparent from the regression lines for each group. Secondly, the output variation not explained by differences in property rights quality seems more significant for the middle income group of countries. This is consistent with the view that these countries, in particular, may be affected by additional problems of development, not included in the current framework.

We conclude this section by providing an additional test for our model, based on the case study by Djankov et all (2008). They construct a measure for the efficiency of debt enforcement in 88 countries, by presenting insolvency practitioners in each country with the same questionnaire of a defaulting firm. Their debt enforcement measure covers several aspects related to time, cost, and the likely disposition of the assets in case of default. For our purposes, we are interested in their measure for the recovery rate (cents per dollar), which represents the part of the debt that the financial intermediary can recover from the firm in case of default. This is a reflection of the parameter λ that in our model characterizes

[14]While we have used a slightly higher threshold than the IMF or the World Bank (given that we do not have any low-income country in our sample), our classification is broadly consistent with theirs.

the quality of property rights. We rescale the recovery rate by its US level, and feed it to our model. The predictions for GDP per worker are displayed in Figure 10. The recovery rate compiled by Djankov et all (2008) reinforces the available evidence for a strong correlation between the quality of property rights and output. Figure 11 displays the country data and model predictions for the average firm size, and provides empirical support to the view that firm size is positively correlated to the quality of property rights.

We keep in mind that their case study referred to a medium firm (from a specific industry), and their findings may not be fully representative for the economy as a whole, in particular for larger firms, which although less in number, account for a significant share of total output[15]. Nonetheless, this exercise reinforces the conclusions of this section with new empirical evidence and renders additional testimony for the significant explanatory power of our model, based on the differences in property rights and the adverse selection mechanism.

7. Model with a Self-funding sector

In our model the differences in the quality of property rights can explain a significant part of the output variation but only a small fraction of the TFP variation across countries. Like most neoclassical literature on growth, we looked at the economy in the aggregate and we consider that all the production is realized by the manufacturing sector. But this approach has a limitation: Rajan and Zingales (1998) have shown that different industries have different dependence on external finance. The manufacturing sector, which is characterized by large scale of operation, is more vulnerable to financial marker imperfections than other sectors where the scale of operation are not that large. As Buera, Kabosky and Shin (2011) pointed out "sectors with larger scales of operation (e.g. manufacturing) have more financing needs, and are hence disproportionately vulnerable to financial frictions". Buera, Kabosky and Shin (2011) incorporate to the model this idea of the special vulnerability of sectors with large scale by introducing two sectors with different scale of operations, namely manufacture and services (characterized by different fixed costs, as in Erosa and Hidalgo, 2008). Another approach is the one by Midrigan and Xu (2012), where entrepreneur may chose two type of technologies: one that do not uses capital and consequently do not requires any outside finance, in this sense is a self-funding technology, and another technology

[15]The authors aknowledge that for large firms the bankrupcy procedure is likely to be politicised (especially in developing countries).

which uses capital and that consequently requires external finance and is affected by financial frictions.

Our approach follows the same spirit that these previous contributions: there is two technologies, the first tecnology is the one introduced in the benchmark model characterized by fixed cost which implies large scale of operation and requires external fund; the second tecnology is characterized by a smaller scale of operation (like Buera, Kabosky and Shin, 2011 and Erosa and Hidalgo, 2008) and by being self-funding and not requiring external funds (like Midrigan and Xu, 2012). We will refer as large scale technology to the first technology and self-funding technology to the second one.

We identify the self-funding technology in our calibration as the agriculture. The reasons for that agriculture is as a sector which is less vulnerable to financial frictions due to several factors: i) agriculture is characterized by small scale of operation, ii) The type of organization that agricultural firms has, which is characterized by self-employment and family and individual business, can be more easily adapted to self-funding than other type of organizations more common in other sectors, namely in manufacturing, iii) agriculture has more weight in developing countries than in developed ones.

The average firm size measured by the number of employees according with the American Census Bureau 2007 is about 55 in manufacture and 14 in services. According with 2007 Agriculture Census[16] and using the same methodology as the one used in calculate the average firm size in manufacture and services, the firm size measure by the number of employees is 0.41 in agriculture, smaller than one. Thus, the average firm size in services is 34 times larger than agriculture and in manufacturing is 133 times larger. The reason for such small size is that only the 9% of farms hire workers. This means that self-employment is the main source of labor in agriculture. Furthermore, the vast majority of farms, the 86%, are organized as familiar or individual business. This type of organization and the small scale of operation suggest that firms in the agricultural sector (farms) are characterized by self-employment and can be more easily self-funding and less vulnerable to the financial frictions than other sectors, like manufacturing.

Another reason to consider agriculture as a sector that is less vulnerable to

[16]Since Agriculture is clearly characterized by seasonal employment, we have consider only hired workers for more than 150 days a year on the farm. If it is consider the total amount of hired workers (included those with less than 150 days per year in the farm) then the firm size in agriculture would be still vey small, on average 1.2 workers per farm, and the self employment still very important, only the 22% of the farms would hire workers.

financial friction is the higher weight that this sector has in less developed countries as table *** shows. The percentage of employment in agriculture in countries with less that the 40% of the US per capita GDP is on average 24%, while in countries with a per capita GDP larger than the 40% of US per capital GDP is smaller than the 4%. If we consider only countries with less than 20% of US per capita GDP, the average percentage of employment in agriculture increases up to 32%. Thus, there is a very clear pattern in the data: the weight of agriculture is more important in poor countries than in rich ones. As Rajan and Zingales (1998) pointed out, industries with more needs of extemal finance develop disproportionately faster in countries with more-developed financial markets. Thus, following the same logic, agriculture which has more weight in countries with weaker property rigths and higher financial friccions should be a sector with less financial needs than others.

The above considerations motivate us to propose in this section an extension of our model including a self-funding technology, which is not as affected by weak property rights as other technologies. To be more precise, for the sake of simplicity we will consider that self-funding technology does not need financial intermediation and, consequently, is not affected by the quality of property rights. We will use the agricultural sector as a proxy for this self-funding technology.

In this section we will consider that in order to produce goods there are two types of technologies, a large scale one, which is exactly as the one described in the main model, and another technology that will be called self-funding technology. The basic difference between large scale and self-funding technology is that the use of large scale technology requires financial intermediation, while the self-funding technology can be financed directly by households, and, consequently does not need financial intermediation. Since, in our model, property rights affect the economy via the financial contract, they will concern firms that use financial intermediation, that is, those that use large scale technology. However, there will be no mechanism via which property rights directly affect the firms using self-funding technologies. We assume that the self-funding technology is characterized by the following production function, which uses labor l_{sf}, capital k_{sf} and land z_{sf} to produce a quantity y_{sf} of the consumption good:

$$y_{sf} = \Gamma_{sf} \left(l_{sf}\right)^{\eta} \left(k_{sf}\right)^{\alpha} (z_{sf})^{1-\alpha-\eta}$$

We have included a specific factor in the self-funding sector: land. The reason is smoothing the reallocation of resources from the self-funding to the large scale sector. Otherwise, there would be a threshold level of property rights above which the self-funding technology is not used and below which the large scale technology is not used. The self-funding technology will employ labor and capital by maximizing profit, according to the FOCs:

$$
\begin{aligned}
\eta \frac{\Gamma_{sf}\,(l_{sf})^{\eta}\,(k_{sf})^{\alpha}(z_{sf})^{1-\alpha-\eta}}{l_{sf}} &= w \Leftrightarrow \eta \frac{y_{sf}}{l_{sf}} = w \\
\alpha \frac{\Gamma_{sf}\,(l_{sf})^{\eta}\,(k_{sf})^{\alpha}(z_{sf})^{1-\alpha-\eta}}{k_{sf}} &= (r+\delta) \Leftrightarrow \alpha \frac{y_{sf}}{k_{sf}} = \delta + r
\end{aligned}
$$

We will assume that capital goods are only produced by the large scale sector. Given that the relative share of this sector will diminish with the quality of property rights, as property rights drop, at some point the large scale sector will only be producing capital for the self-funding sector. This creates a bound for the self-funding sector, in the sense that, once the quality of property rights drops enough, self-funding output will be limited by the amount of capital produced by the large scale sector.

7.1. Calibration of self-funding technology

As explained above, since information about informal economy is very imprecise, and one of the most salient facts about sectorial differences among developing and developed countries is the much higher weight of agriculture in developing countries than in developed countries, we will use the agricultural sector as a proxy for the self-funding technology. We used exactly the same parameters as in our benchmark calibration for the large scale technology, preferences and depreciation of capital.

To calibrate the self-funding production function, we take the US economy as a benchmark, considering agriculture as the self-funding sector. We draw from measures of factor income shares of Herrendorf and Valentinyi (2008), and from statistics on agriculture value added (% of GDP) as published by the World Bank and OECD. Mapping the model to the industry data is not straightforward, given that our model only uses labor, capital and land to produce the final output, while the industries in the data use intermediate inputs, capital, and labor to produce intermediate inputs for other industries and final output. Given that our production function has constant returns to scale, it suffices to fix the share of labor and the share of capital (in our framework the capital consists of structures and equipment, excluding land, which is a separate factor supplied in a fixed amount).

Herrendorf and Valentinyi (2008) have proposed a mapping between the Input-Output data and the most common two sector models. Among their findings, they noted that if we take the land share out, then the remaining capital share in agriculture is close to the economy-wide average. Thus, we take the capital share in the self-funding technology (agriculture) to be the same as in the large scale technology, 0.36. We then use Herrendorf and Valentinyi (2008)' income share of labor in agriculture of 0.59[17]. Finally, the constant factor $\Gamma_{sf}(z_{sf})^{1-\alpha-\eta}$ encompassing the total factor productivity in agriculture and the fixed stock of land is deduced from matching the US agriculture value added (% of GDP), as published by the World Bank and OECD. For consistency with the rest of the paper, we used the average for years 1995-2010 (of 1.3%).

7.2. Results with self-funding technology

The model with self-funding technology generates a stronger variation of TFP, as shown in Figure 12. When property rights drop, this affects the large scale technology, but not the self-funding technology; this generates a reallocation of resources from the large scale sector to the self-funding one. More precisely, when property rights worsen, the highly productive establishments using large scale technology become inefficiently small, using less capital and workers than the optimum. Then, the large scale technology generates a lower output. The novelty of this exercise is this reallocation of resources which takes place from the large scale technology to the self-funding one: as the quality of property rights diminishes, the self-funding sector employs relatively more labor and capital. On the aggregate, given that the self-funding technology is less productive than the large scale one, the economy-wide TFP drops much more than before.

This is closer to what cross country data suggest. To see this we plotted in Figure 13 the TFP against the output per worker: the line shows the prediction of the model with self-funding technology, while the rhombuses represent data for 190 countries from PWT for the year 2005, the base year of the PWT database

[17] Here we have two options, as they provide the factor income shares at sector level both at producer and purchaser prices. The difference is that when factor shares are computed at purchaser prices, the distribution costs are included as part of the sector output. The result in this case is a lower capital share for agriculture, which could be intuitively explained by the lower capital intensity of distribution services compared to agriculture. Given that the PPP adjusted data on GDP per worker that we use to compare output among countries is based on purchaser prices, we decided to follow the same line, resulting in a labor income share of 0.59. However, we have experimented both options, and the results do not change by much.

(both TFP and output are normalized by the US level). TFP is computed, as in the rest of this paper, by the equation 6.1a. We compute the capital using the perpetual inventory method, where the depreciation rate is the one in our model and we consider the initial capital as the one predicted by the neoclassical model at the steady state. That is:

$$\begin{aligned} k_{\text{initial period}} &= \left(\frac{\alpha}{\delta+\rho}\right) y_{\text{initial period}} \\ k_{t+1} &= i_t + (1-\delta)k_t \end{aligned}$$

where we use as the initial period the first year available in the PWT for each country. A convincing pattern emerges, with TFP varying across countries in a clear one-to-one relationship with the output per worker, both in the data and in the model, in accordance with the implications of the neoclassical growth framework. Quite remarkably, this relationship holds for the 190 countries in the PWT sample, covering countries with very different levels of output.

8. Conclusion

Our quantitative experiments attempt to contribute to the literature analyzing the relationship between the quality of property rights enforcement, financial development and economic development. We are particularly interested in the aggregate effects of adverse selection. The contribution of this paper is to model property rights enforcement and adverse selection in an intuitive way which, at the same time captures several features of the real world so as to be suitable for a quantitative analysis.

Following the literature, we introduce the imperfect quality of property rights in the context of the credit market. Firms need to borrow in order to produce and the quality of property rights is given by the variable ability of financial intermediaries to recover the credit from firms. Firms have different productivity which will affect their size. We introduce asymmetric information about this ability, and this induces an adverse selection effect. Our main objective is to assess the effects on the aggregate economy. In our experiments, we focus on output, productivity and firm size. To keep computations tractable, we consider two types of managers: with high and low productivity. In this way, we have a general equilibrium model which encompasses two capital market imperfections (imperfect property rights and adverse selection) and which is suitable for numeric experiments.

We calibrate our model and perform some experiments by varying the degree of property rights enforcement so as to match the average firm size in a sample of 30 middle and high income countries for which comparable data is available. Through differences in the quality of property rights enforcement, our model is able to explain to a great deal of the differences in output between countries. When adverse selection is taken into account, the explanatory power of the model is significantly increased: the experiments indicate that alleviating informational problems can raise output by up to 20%. This suggests that informational problems have important effects on the aggregated output and policies targeted at this issues may be worth pursuing.

The model is able to generate some variation in productivity too, but there remains variation in the data which is not accounted for. To address this, one could enrich the model in several ways. We performed one experiment by including a self-funding technology, which in less developed countries is more important and which could be expected to be less affected by the quality of property rights. The quantitative effects on TFP proved to be significant. The model could also be extended, for instance, by taking into account the effect of property rights on the process of technology accumulation. We leave this however for future research.

Finally, it could be useful to put our work in the broader context. Throughout the analysis, one question that emerges quite naturally is how and why countries end up with different level of property rights enforcement? Acemoglu et all (2005, 2011) provided some examples of how economic institutions, which shape economic outcomes, are determined by political power, which is in turn determined by political institutions and the distribution of resources in society. They identified constructing formal models incorporating and extending their theory to be an important task ahead.

At the same time, the recent economic and financial developments affecting the world economy suggest that considering the growing interactions among national economies may also contribute to a better understanding of the phenomena studied in this paper. All these efforts are likely to contribute to a better understanding of the economic inequalities between countries.

9. Appendix

9.1. Proof proposition 4.2

We showed that the *NDC* is equivalent to $k_j \leq k_{1,j}(\lambda)$. We denoted the level of capital which maximizes the manager payoff as $k_{j,*}$. *NDC* is binding if and only if $k_{j,*} \geq k_{1,j}(\lambda)$. But $k_{j,*}$ is $\underset{k}{argmax}\ \pi(\theta_j, k) - (r+\delta)k$, so by FOC we get $k_{j,*} = [\theta_j(\frac{\alpha}{r+\delta})^{1-\beta}\left(\frac{\beta}{w}\right)^{\beta}]^{\frac{1}{1-\alpha-\beta}}$. Also from definition of $k_{1,j}(\lambda)$ we get that $k_{1,j} = [\theta_j(\frac{\lambda(1-\beta)}{r+\delta})^{1-\beta}\left(\frac{\beta}{w}\right)^{\beta}]^{\frac{1}{1-\alpha-\beta}}$. Then $k_{j,*} \geq k_{1,j}(\lambda)$ is equivalent to $\frac{\alpha}{1-\beta} \geq \lambda$, so *NDC* is binding if and only if $\lambda \leq \frac{\alpha}{1-\beta}$.

we show that in case 1, $\lambda \leq \frac{\alpha}{1-\beta}$, the *RC* is also binding and the equilibrium menu of contracts is $k_g = k_b = k_{1,b}(\lambda)$.

If NDC is binding then $k_b = k_{1,b}$:

$$
\begin{aligned}
&\forall k > k_{1,b} \\
&\pi(\theta_b, k_b) - (r+\delta)k_b = \pi(\theta_b, k_{1,b}) - (r+\delta)k_{1,b} = \\
&(1-\lambda)\pi(\theta_b, k_{1,b}) < (1-\lambda)\pi(\theta_b, k)
\end{aligned}
$$

Thus $k_{1,b}(\lambda) = k_2(\lambda, k_{1,b})$. Since If *NDC* is binding $k_b = k_{1,b} \leq k_{b*}$. It is follows from the assumption that $\frac{\partial^2\pi(\theta,k)}{\partial k\partial\theta} > 0$ that $k_{g*} > k_{b*}$. Thus $k_2(\lambda, k_{1,b}) = k_{1,b}(\lambda) \leq k_{b*} < k_{g*} \Rightarrow k_g = k_2(\lambda, k_{1,b}) = k_{1,b}(\lambda)$.

will prove lemma 3 in four steps:

Step 1: *Given r, there is $\lambda_2 > \lambda_1$ such that $\forall\lambda \in (\lambda_1, \lambda_2)$ we have $k_b = k_{b*} < k_g = k_2 < k_{g*}$.*

It follows from definition of λ_1 that when $\lambda = \lambda_1$ $k_{b*} = k_2 = k_{1,b} < k_{1,g}$ and $k_2 = k_{b*} < k_{g*}$. It follows from the definitions of k_2 and $k_{1,g}$, and Implicit Function Theorem that both k_2 and $k_{1,g}$ are continuous increasing functions of λ for $\lambda > \lambda_1$:

$$
\begin{aligned}
\frac{\partial k_2}{\partial\lambda} &= \frac{\pi(\theta_b, k_2)}{(1-\lambda)\frac{\partial\pi(\theta_b,k_2)}{\partial k}} > 0 \\
\frac{\partial k_{1,g}}{\partial\lambda} &= -\frac{\pi(\theta_g, k_{1,g})}{\lambda\frac{\partial\pi(\theta_g,k_{1,g})}{\partial k} - (r+\delta)} = -\frac{\pi(\theta_g, k_{1,g})}{\lambda\left[\frac{\partial\pi(\theta_g,k_{1,g})}{\partial k} - \frac{\pi(\theta_g,k_{1,g})}{k_{1,g}}\right]} > 0
\end{aligned}
$$

Let's define $\lambda_2 = \min\left\{\frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})}, \min\{\lambda \in [\lambda_1, 1] \text{ such that } k_2 = k_{1,g}\}\right\}$, it follows from definition of λ_2 that $\forall\lambda \in (\lambda_1, \lambda_2)$ $k_{1,g} > k_2 > k_{b*} = k_b$ and $k_{g*} > k_2$, therefore $k_g = k_2 < k_{g*}$.

Step 2: *Given r, there is $\lambda_2 > \lambda_1$ such that $\forall \lambda \in (\lambda_1, \lambda_2)$ we have $k_b = k_{b*} < k_g = k_2 < k_{g*}$.*

Let's define $\lambda_3 = \max\left\{\frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})}, \frac{(r+\delta)k_{g*}}{\pi(\theta_g,k_{g*})}\right\}$. It follows from the definition of λ_3 that when $\lambda = \lambda_3$, $\min\{k_{1,g}, k_2\} = k_{g*} = k_g$. Since both $k_{1,g}$ and k_2 are increasing functions of λ, it follows that $\forall \lambda > \lambda_3$, $\min\{k_{1,g}, k_2\} > k_{g*} = k_g$.

Step 3: *If $\pi(\theta, k) = g(\theta)k^\xi$, being $g(.)$ a continuous increasing function and $\xi > 0$, then $\lambda_3 = \lambda_2$.*

In this case

$$
\begin{aligned}
k_{b*} \quad &\Leftrightarrow \quad (r+\delta) = \frac{\partial \pi\left(\theta_b, k_{b*}\right)}{\partial k} = \xi \frac{\pi\left(\theta_b, k_{b*}\right)}{k_{b*}} \\
k_{g*} \quad &\Leftrightarrow \quad (r+\delta) = \frac{\partial \pi\left(\theta_g, k_{g*}\right)}{\partial k} = \xi \frac{\pi\left(\theta_g, k_{g*}\right)}{k_{g*}}
\end{aligned}
$$

This implies that:

$$
\lambda_1 \equiv \frac{(r+\delta)k_{b*}}{\pi\left(\theta_b, k_{b*}\right)} = \xi = \frac{(r+\delta)k_{g*}}{\pi\left(\theta_b, k_{g*}\right)}
$$

Therefore when $\lambda = \lambda_1$ $k_{1,g} = k_{g*} > k_{b*} = k_2$. Therefore, for $\lambda > \lambda_1$, the IC1 is not binding (Neither for the good nor the bad type). Thus, if $\lambda \in \left(\lambda_1, \frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})}\right]$ $k_2 \le k_{g*} < k_{1,g} \Rightarrow \lambda_2 = \frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})}$. Furthermore, if $\lambda = \lambda_2 = \frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})}$ $k_2 \le k_{g*} < k_{1,g} \Rightarrow \pi\left(\theta_g, k_{g*}\right) - (r+\delta)k_{g*} > (1-\lambda_2)\pi\left(\theta_g, k_{g*}\right) \Rightarrow \lambda_2 = \frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})} > \frac{(r+\delta)k_{g*}}{\pi(\theta_g,k_{g*})} \Rightarrow \lambda_3 = \frac{\pi(\theta_g,k_{g*})-\pi(\theta_b,k_{b*})+(r+\delta)k_{b*}}{\pi(\theta_g,k_{g*})} = \lambda_2$.

Step 4: *Computing λ_2 and $k_2(\lambda, k_{b*})$ for Case 2, $\lambda \ge \frac{\alpha}{1-\beta}$.*

We have seen that in Case 2, when $\lambda \ge \frac{\alpha}{1-\beta}$, the *NDC* is not binding so we do not need worry for it.

Going to *RC*, we showed in section 4.1 that it is equivalent to $k_g \le k_2(\lambda, k_b)$, where $k_2(\lambda, k_b)$ is the value of k_g such that the *RC* holds with equality:

$$
\pi\left(\theta_b, k_b\right) - (r+\delta)k_b \;=\; (1-\lambda)\pi\left(\theta_b, k_2(\lambda, k_b)\right)
$$

As $k_2(\lambda, k_b)$ is an increasing function of k_b on $[0, k_{b*}]$, the optimal contract will be $k_b = k_{b*}$ and $k_g = \min\{k_2(\lambda, k_{b*}), k_{g*}\}$. Now let us find $k_2(\lambda, k_{b*})$.

We use the fact that $\frac{\alpha}{1-\beta}\frac{\pi(\theta_b,k_{b*})}{k_{b*}} = r+\delta$ so we substitute $(r+\delta)k_{b*}$ with $\frac{\alpha}{1-\beta}\pi\left(\theta_b, k_{b*}\right)$ in **??** and then by using also that $\pi\left(\theta_b, k\right) = \left[\theta_b k^\alpha (\frac{\beta}{w})^\beta\right]^{\frac{1}{1-\beta}}(1-\beta)$ we get that:

$$k_2(\lambda, k_{b*}) = k_{b*}(\frac{1-\alpha-\beta}{(1-\lambda)(1-\beta)})^{\frac{1-\beta}{\alpha}}$$

Then the RC is binding if and only if $k_2(\lambda, k_{b*}) \leq k_{g*}$ which is equivalent to

$$\lambda \leq 1 - (\frac{\theta_b}{\theta_g})^{\frac{\alpha}{(1-\beta)(1-\alpha-\beta)}}(\frac{1-\alpha-\beta}{1-\beta})$$

Then $\lambda_2 \equiv 1 - \left(\frac{\theta_b}{\theta_g}\right)^{\frac{\alpha}{(1-\beta)(1-\alpha-\beta)}} \left(\frac{1-\alpha-\beta}{1-\beta}\right)$.

9.2. Proof proposition 6.1

We defined TFP as $\left(\frac{Y}{K^\alpha}\right)^{\frac{1}{1-\alpha}}$. We will show that $\frac{\partial \frac{Y}{K^\alpha}}{\partial \lambda} > 0$. The most difficult case is $\lambda \in (\lambda_1, \lambda_2)$. When $0 < \lambda \leq \lambda_1$ the formulas are much simpler and when $\lambda \geq \lambda_2$ we are in the perfect information case.

Let us start with some simplifying notation. Denote $s = (\frac{1-\alpha-\beta}{(1-\lambda)(1-\beta)})^{\frac{1-\beta}{\alpha}}$, $f = \frac{\theta_b}{\theta_g}$, $a_1 = \nu s + (1-\nu)$, $a_2 = \nu f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}} + (1-\nu)$.

Note that $\lambda \in (\lambda_1, \lambda_2)$ implies

$$1 < s < f^{\frac{1}{1-\alpha-\beta}} \tag{9.1}$$

. Then we get:

$$TFP = \frac{Y}{K^\alpha} = \frac{\frac{1-l}{\varepsilon} a_2 (\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\beta}} (\frac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}}{\left\{\frac{1-l}{\varepsilon} a_1 (\frac{\alpha}{r+\delta})^{\frac{1-\beta}{1-\alpha-\beta}} (\frac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}\right\}^\alpha} = \frac{[(1-\beta)a_2 - \alpha a_1]^{1-\alpha-\beta} a_2}{(a_2 - \alpha a_1)^{1-\alpha} a_1^\alpha} const(\lambda).$$

As $s(\lambda)$ is strictly increasing in λ on $(0,1)$, it is equivalent to show that TFP is increasing in s. Due to the size of the expression of TFP when substituting for a_1 and a_2, we put it in *Mathematica*, simplify it, then take its derivative w. r. to s, simplify it.

The last step is to show that all the factors of the resulting product are positive

$-s + f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}} > 0$ and $1 + (-1+s)v > 0$ directly from 9.1

$\alpha(-1+\nu-s\nu) - (-1+\beta)[1 + (-1 + f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu] > 0 \Leftrightarrow (1-\alpha-\beta)(1-\nu) - as\nu + (1-\beta)\nu f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}} > 0$, true by using $1-\alpha-\beta > 0$ and 9.1.

The last inequality to prove is:

$$\frac{\alpha^2[1+(-1+s)\nu](-1+\beta+\nu-\beta\nu+\beta f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}}\nu)}{(-1+\beta)s} + \frac{\alpha\{-2+\beta+[2+\beta(1+s)]\nu\}[1+(-1+f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu]}{s} +$$
$$+ \frac{(-1+\beta)(-1+\nu)[1+(-1+f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu]^2}{s+(-1+s)s\nu} > 0.$$

We simplify it in Mathematica using the constraints $\{s > 1, \beta < 1, \nu > 0, \nu < 1\}$ and we get:

$[\alpha+\alpha(-1+s)\nu]^2(-1+\beta+\nu-\beta\nu+\beta f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}}\nu)+\alpha(-1+\beta)\{-2+\beta+[2+\beta(1+s)]\nu\}[1+(-1+s)\nu]+(-1+\beta)^2(-1+\nu)[1+(-1+f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu]^2 < 0$ which is equivalent to:

$(-1+\nu)\{\alpha+\alpha(-1+s)\nu+(-1+\beta)[1+(-1+f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu\}^2+(-1+\alpha+\beta)[1+(-1+s)\nu]^2[1+(-1+f^{\frac{1}{1-\beta}} s^{\frac{\alpha}{1-\beta}})\nu] < 0$, which is true as $-1+\nu < 0$ and $-1+\alpha+\beta < 0$.

9.3. Calibration details

We want our model at Steady State to match the Firm Size Distribution (FSD) for the U.S. At the same time, we want to use the FSD in other countries to assign a value to the enforcement parameter λ for those countries. For the U.S., the economic census provides probably the best available data on FSD (together with the County Business Patterns).

For the calibration, we consider that large firms in the data are those which have an amount of workers above the sector average which is 41.9. From Table 9.1 it can be seen that the size classes which have on average establishments with more than 41.9 are those size classes with more than 50 employees. These will be considered to be the larger (and more productive) group in our economy. This implies a share of large firms in the total number of firms, $\nu = 0,16$.

We set β such that the model matches the portion of output and labor of firms with high productivity. According with the model:

$$\begin{aligned}
\frac{\nu l_g}{E(l)} &= \frac{\left[\nu(1-\alpha-\beta)+\beta\frac{\nu y_g}{E(y)}\right]}{(1-\alpha)} \Leftrightarrow \\
\beta &= \frac{\left[\frac{\nu l_g}{E(l)}-\nu\right](1-\alpha)}{\frac{\nu y_g}{E(y)}-\nu} = \frac{\left[\frac{l_g}{E(l)}-1\right](1-\alpha)}{\frac{y_g}{E(y)}-1} = 0,58
\end{aligned}$$

We chose ε such that the model match the average firm size:

$$E(l) = \frac{(1-\alpha)\varepsilon}{1-\alpha-\beta} \Leftrightarrow \varepsilon = \frac{(1-\alpha-\beta)}{1-\alpha}E(l) = 3,96$$

We set (θ_g/θ_b) such that the portion of output that produce firms with high

Size class	Number of establishments	Number of employees	Average number of employees per establishment	Value Added	Average Value Added per employee
All	350.828	14.699.536	41,9	1.887,7	128,4
1_4	141.992	279.481	1,97	21,6	77,3
5_9	49.284	334.459	6,79	28,3	84,6
10_19	50.824	702.428	13,82	58,0	82,6
20_49	51.660	1.615.349	31,27	142,8	88,4
50_99	25.883	1.814.999	70,12	181,7	100,1
100-249	20.346	3.133.384	154,00	357,4	114,1
250_499	6.853	2.357.917	344,07	297,0	126,0
500_999	2.720	1.835.386	674,77	286,1	155,9
1000_2499	1.025	1.494.936	1458,47	262,0	175,3
2500_	241	1.131.197	4693,76	252,4	223,1

Table 9.1: Descriptive statistics for the establishments in U.S. manufacturing sector (source: the 2002 U.S. Economic Census)

productivity match the data:

$$\begin{aligned}
\frac{\nu y_g}{E(y)} &= \frac{\nu \theta_g^{\frac{1}{1-\alpha-\beta}}}{\nu \theta_g^{\frac{1}{1-\alpha-\beta}} + (1-\nu)\theta_b^{\frac{1}{1-\alpha-\beta}}} = \frac{\nu\left(\theta_g/\theta_b\right)^{\frac{1}{1-\alpha-\beta}}}{\nu\left(\theta_g/\theta_b\right)^{\frac{1}{1-\alpha-\beta}} + (1-\nu)} \Leftrightarrow \\
(\theta_g/\theta_b) &= \left[\frac{(1-\nu)}{\nu}\frac{\frac{y_g}{E(y)}}{1-\frac{\nu y_g}{E(y)}}\right]^{1-\alpha-\beta} = 1,24
\end{aligned}$$

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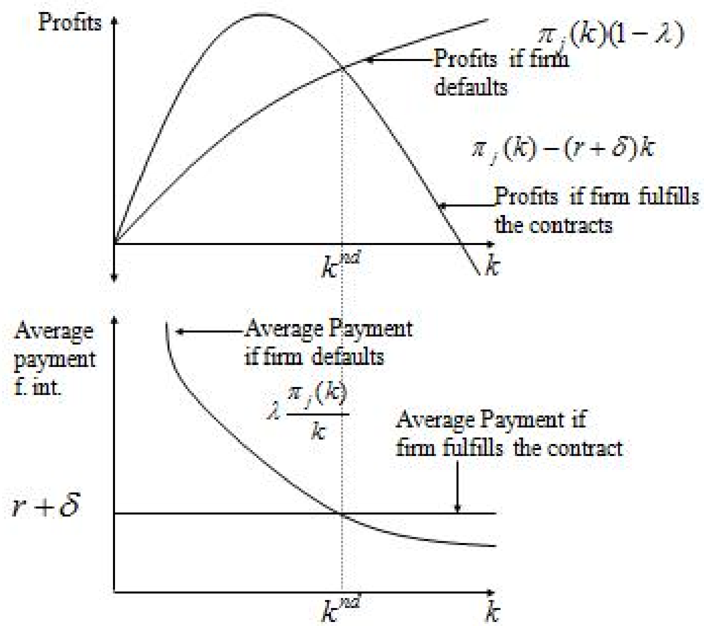

Figure 9.1: The Non-Default Constraint

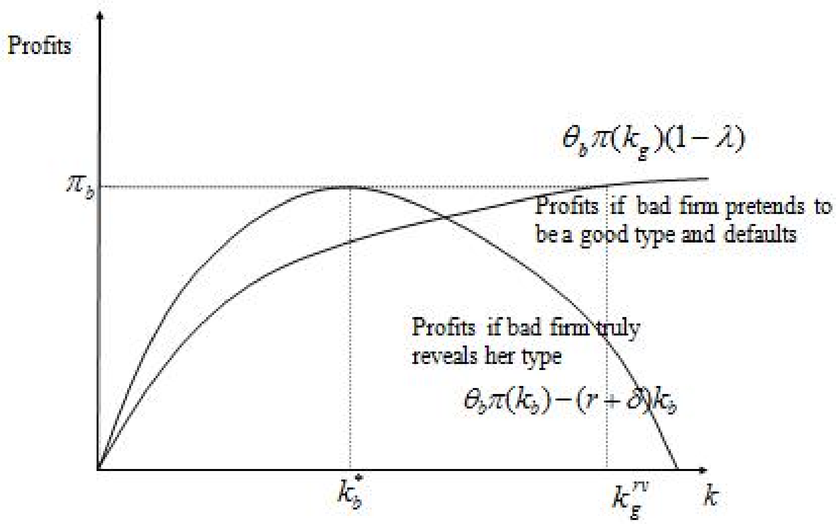

Figure 9.2: The Revelation Constraint

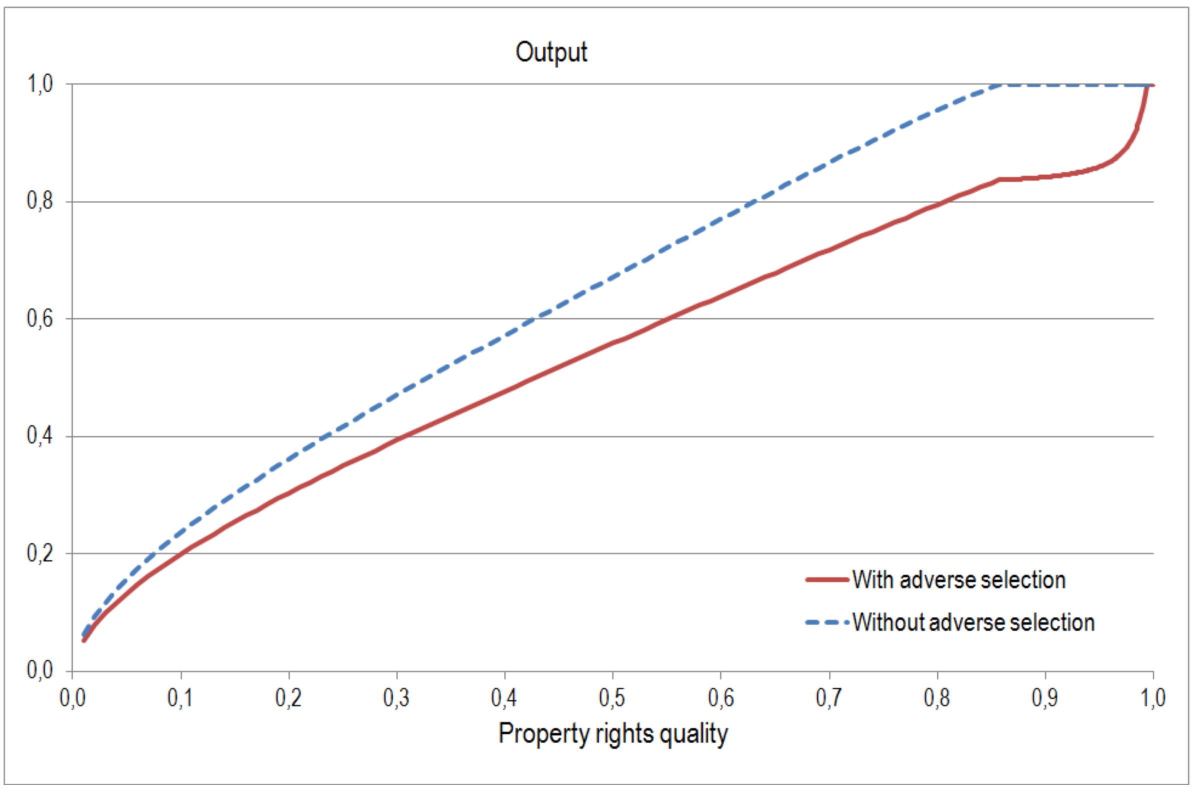

Figure 9.3: Effect of property rights and adverse selection on ouput

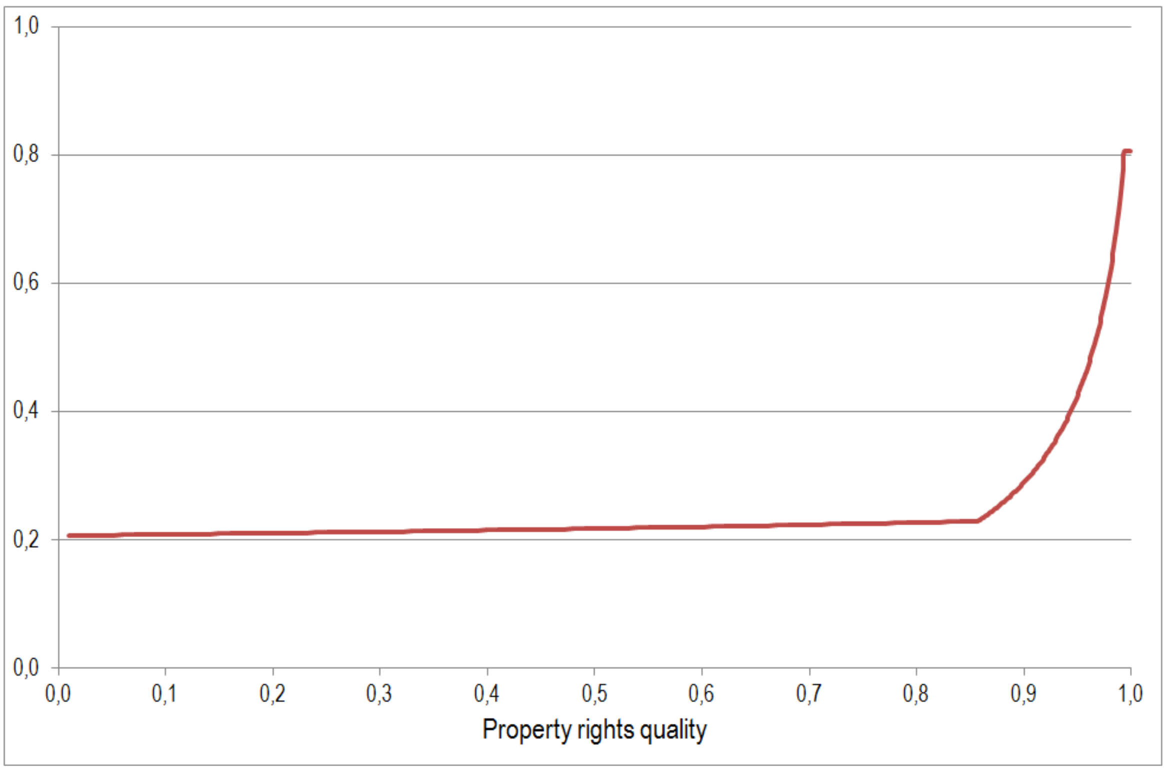

Figure 9.4: Share of labour hired by good firms

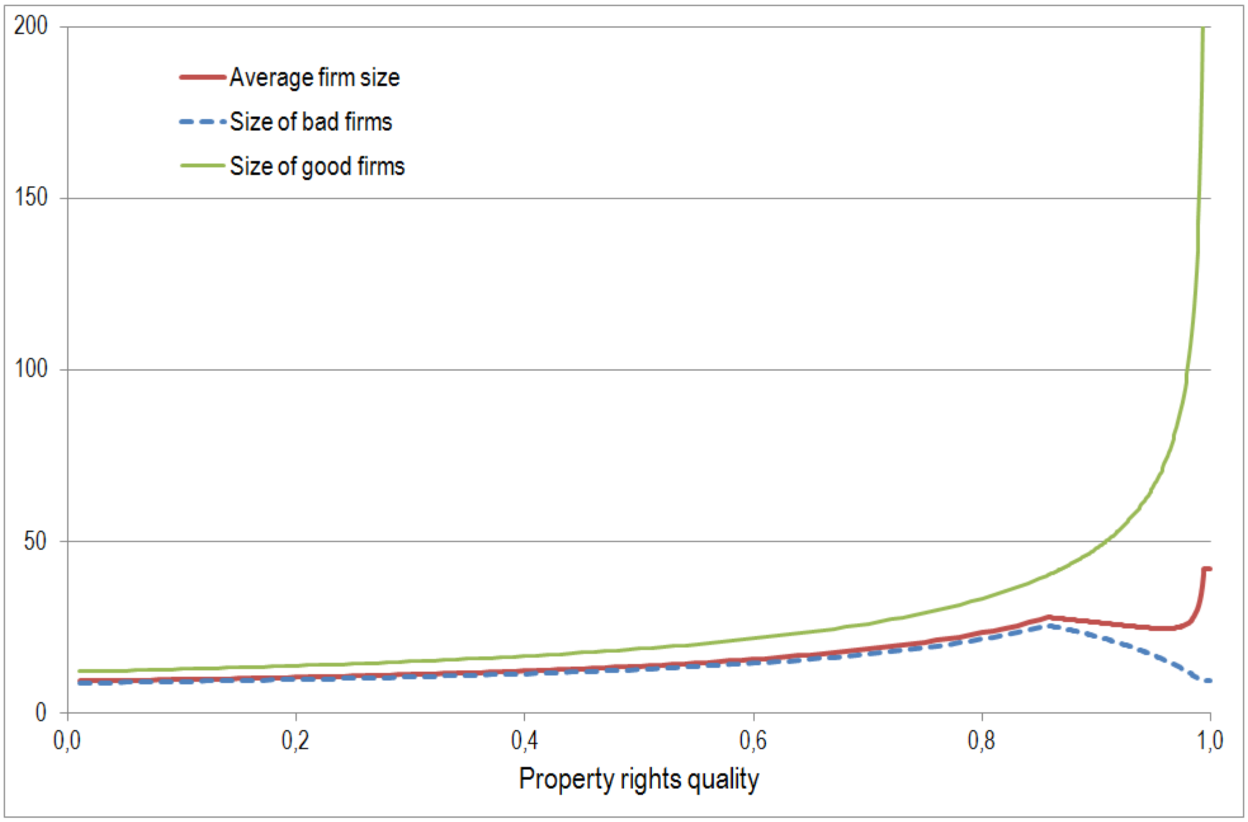

Figure 9.5: Effect of property rights and adverse selection on firm size (no of employees)

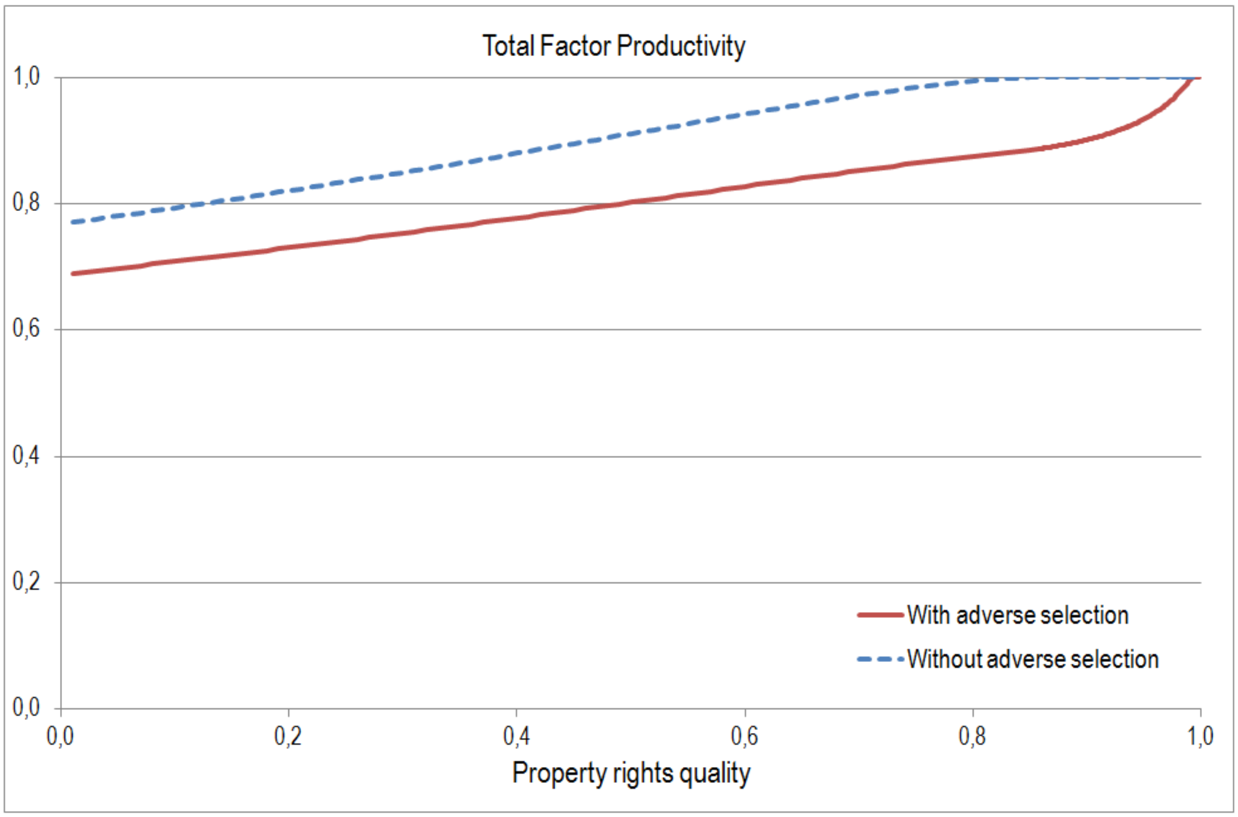

Figure 9.6: Effect of property rights and adverse selection on TFP

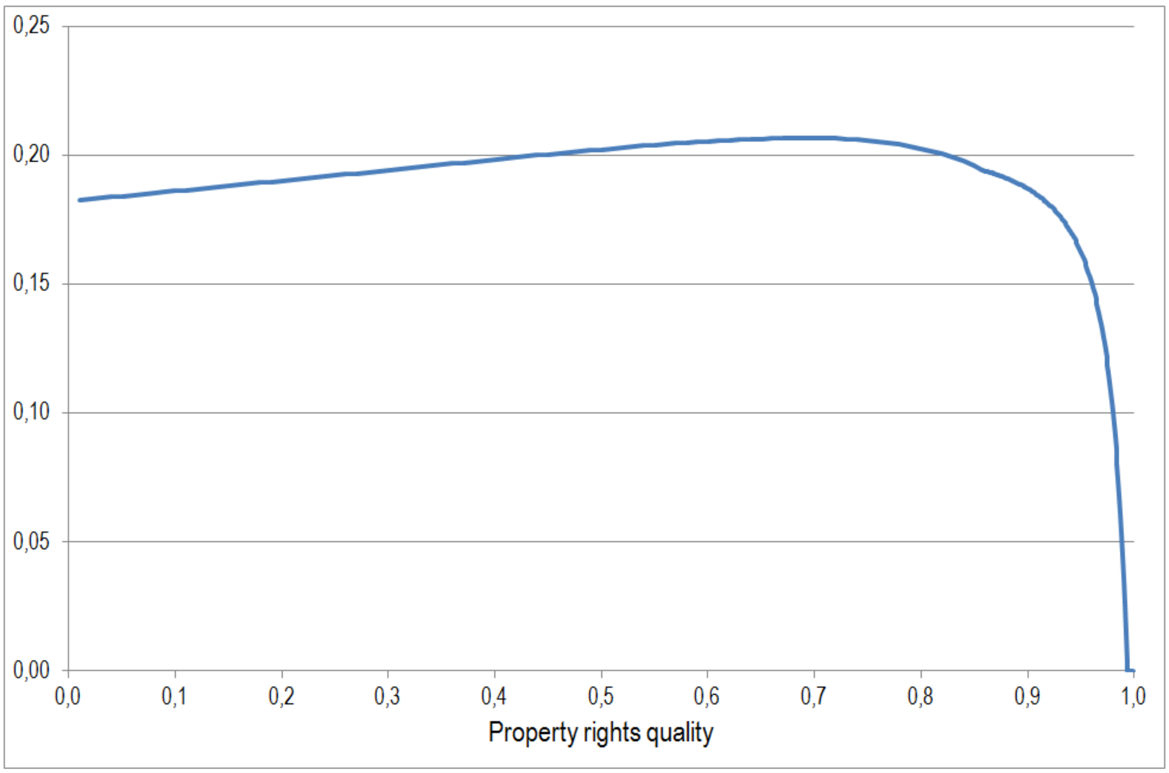

Figure 9.7: Output gain from eliminating adverse selection

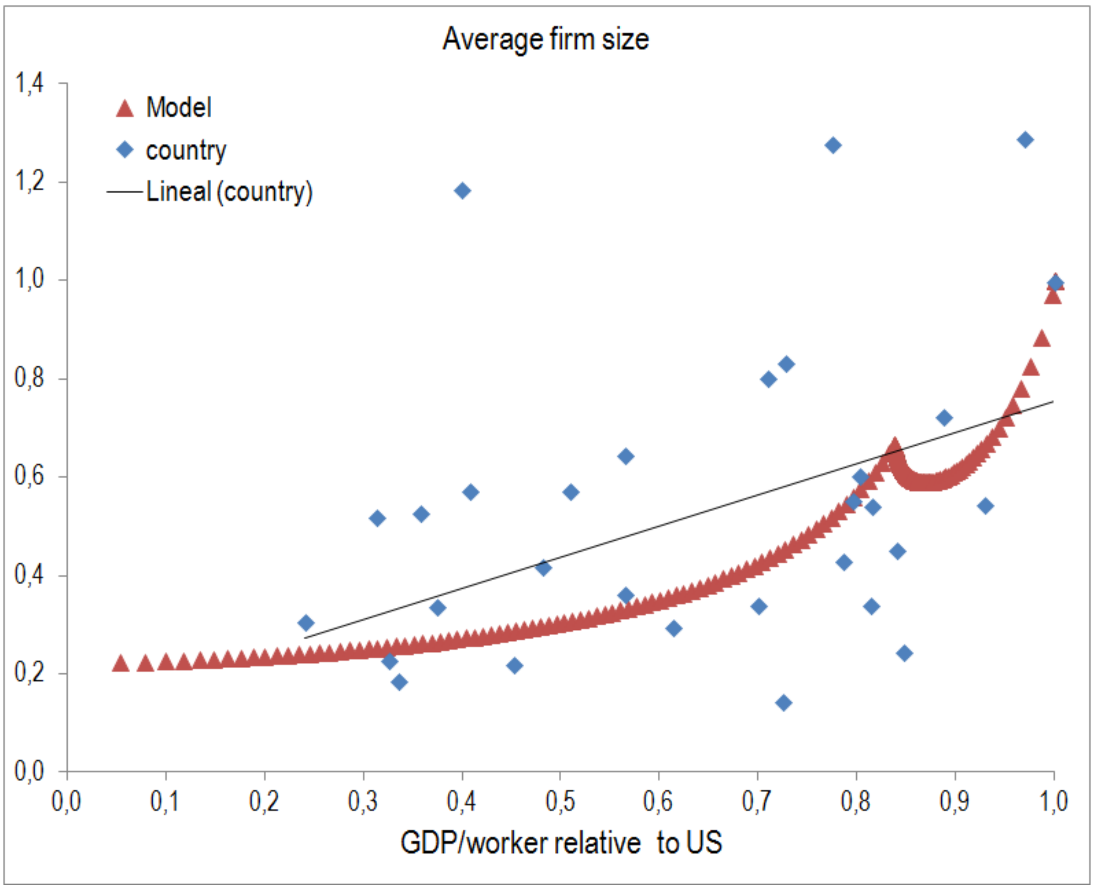

Figure 9.8: Effect of property rights (proxied by firm size) and adverse selection on output: model prediction vs. country data

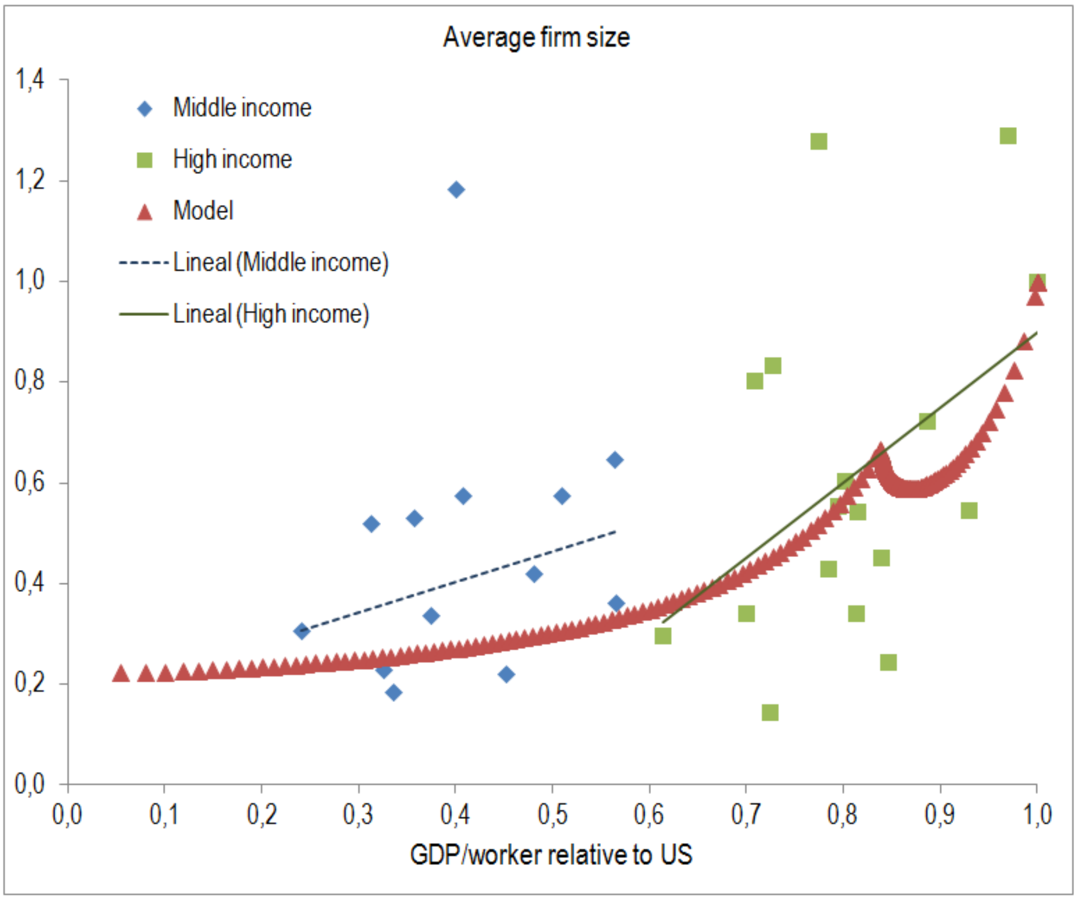

Figure 9.9: Effect of property rights (proxied by firm size) and adverse selection on output: middle income vs. high income countries

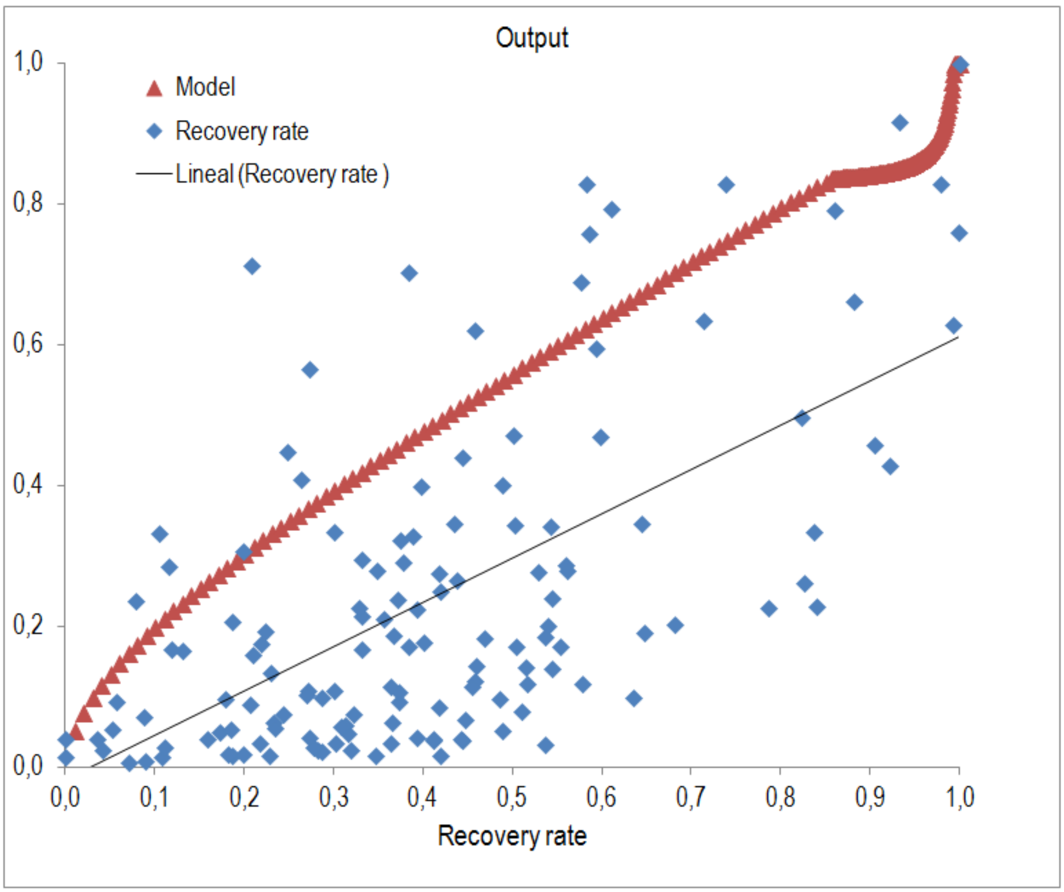

Figure 9.10: Effect of property rights (recovery rate of Djankov et all 2008) and adverse selection on output: Model vs. data

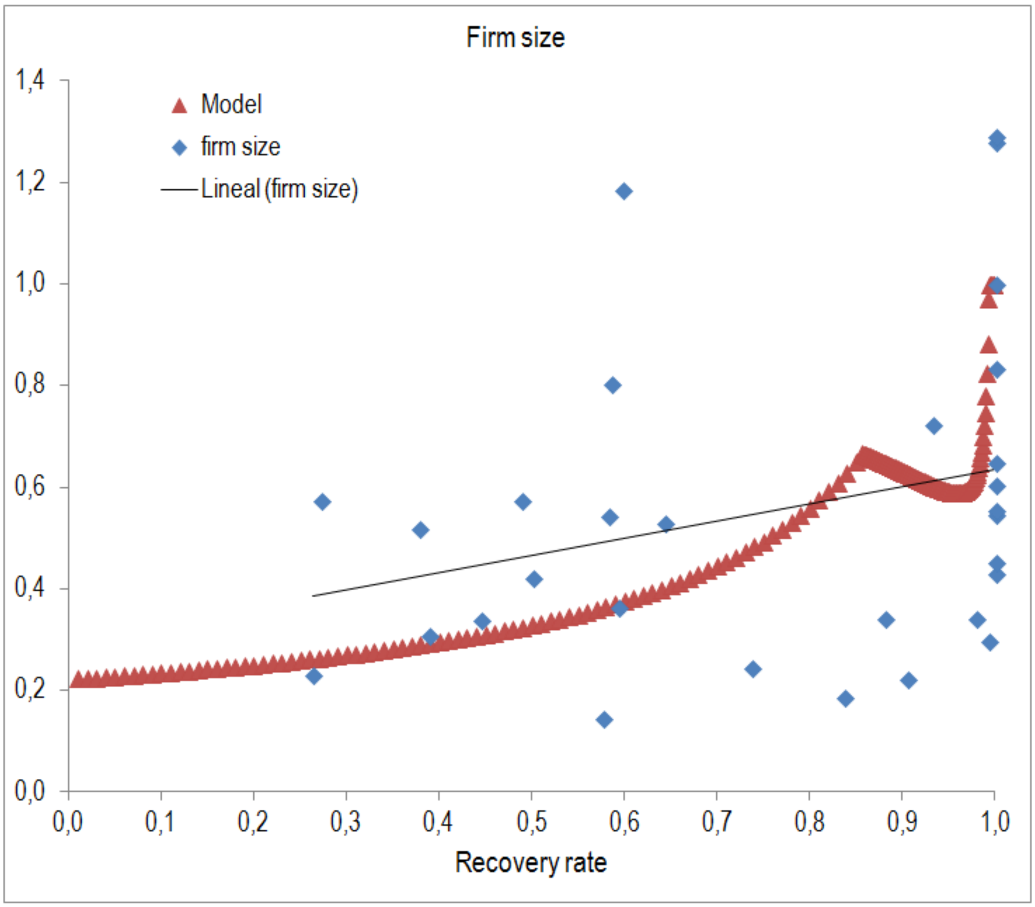

Figure 9.11: Effect of property rights (recovery rate of Djankov et all 2008) and adverse selection on firm size: Model vs. data

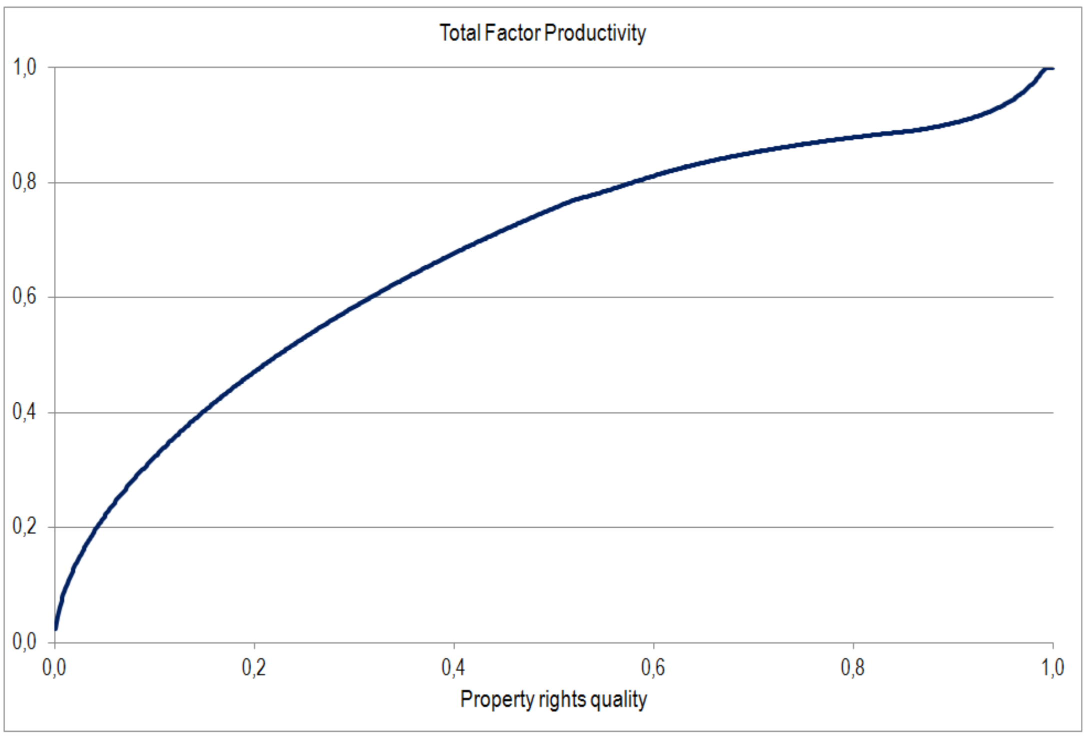

Figure 9.12: Effect of property rights and adverse selection on TFP in the model with traditional technology

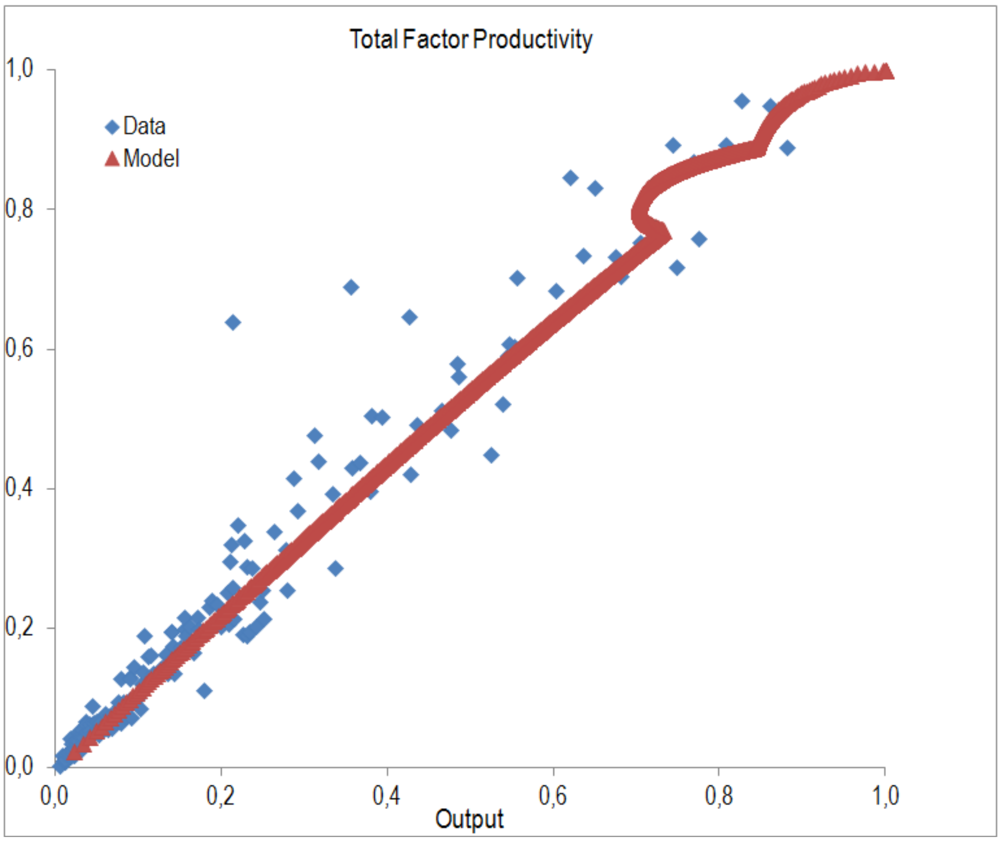

Figure 9.13: TFP vs. output in the data and in the model with traditional technology

Chapter 3

Learning-by-doing, Financial Frictions and TFP

March 20, 2014

Abstract

In this paper we inquire quantitatively whether the ineffective capital allocation across firms generated by financial frictions acts as a barrier that withholds developing countries from adopting more advanced technologies and increasing their TFP. The financial market is affected by imperfect property rights and adverse selection, while firms' current ability in using technology depends on the advancement level of technologies used in the past. The model reveals non-trivial consequences for the technological gap across firms in the economy, which in turn exacerbate the adverse selection problems and significantly amplify the effect on TFP.

1 Introduction

There is a vast literature trying to explain existing differences in levels of income per capita among economies[1]. Why some countries produce so much more per capita then others? Using growth models, documented differences in capital seem not able to account for the observed differences in output (total factor productivity varies greatly across countries)[2]. At the same time, capital market imperfections have figured prominently in the literature as being highly correlated to development[3]. Moreover, the average firm size in developing countries has been found to be often inefficiently small[4]. These observations raise some questions on economic development: How much of the measured differences in output per worker could be explained by taking into account the capital market imperfections? How much of the cross country variation in the average scale of production could be explained by the imperfections in contract enforcement?

These questions prove to be important from several perspectives. Thus, the Organization for Economic Cooperation and Development (OECD) states in its 'Policy for investment framework' that "the ability to make and enforce

[1] This is one of the main questions in development economics.

[2] See for example Prescott (1998), or Hall and Jones (1999).

[3] See for example Goldsmith (1969), McKinnon (1973), Shaw (1973), or King and Levine (1993).

[4] See Tybout (2000).

contracts and resolve disputes is fundamental if markets are to function properly...When procedures for enforcing commercial transactions are bureaucratic and cumbersome, economies rely on less efficient commercial practices... banks reduce the amount of lending because they cannot be assured of the ability to collect on debts... This limits the funding available for business expansion and slows down trade, investment, economic growth and development"[5].

The World Bank devotes important resources to its "Doing Business" project which provides objective measures of business regulations for firms in 183 economies[6]. One dimension of this project covers contract enforcement. The indicators measure the efficiency of the judicial system in resolving a commercial dispute.

These international institutions closely monitor the evolution of contract enforcement in developing economies as they consider that effective commercial dispute resolution has many benefits for development. From a theoretical perspective, this seems justified: Several models have established a qualitative link between contract enforcement, financial development and economic development[7]. Also, econometric studies using cross-country regressions indicate that financial development could have a significant impact on economic development[8]. Our goal is to contribute to this debate by quantifying the effects of limited enforcement on economic development and firm size in the context of a dynamic general equilibrium model.

Below we document the observations we quantify with our model. The first one is that better contract enforcement has a positive effect on financial development which in turn positively affects output. La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997) documented that better law enforcement and accounting standards, positively impact on the development of financial markets. At the same time, many economists (e.g. McKinnon, 1973; Fry, 1995, King and Levine 1993) have discussed the relationship between financial development and output growth[9].

The second empirical fact we aim to quantify refers to the firm size generally displaying a positive correlation with the level of contract enforcement and financial development. Beck, Demirgüç-Kunt and Maksimovic (2006) find that firm size is positively related to financial intermediary development, the efficiency of the legal system and property rights protection[10]. Moreover, a recent study by Garcia-Santana and Ramos (2013) has shown that countries with larger distortions allocate more resources to small unproductive units and that the capacity of the economy to provide credit is crucial to understand the large cross-country differences in the allocation of labor across production units of different size. Their results suggest that this type of distortions is important to

[5] See oecd.org

[6] See www.doingbusiness.org

[7] See, among many others, Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Banerjee and Newman (1993), Khan (2001), Erosa and Hidalgo (2008).

[8] See, among others, Pagano (1993), Levine (1997, 1998).

[9] Other empirical work in this vein includes Gelb (1989), Roubini and Sala-I-Martin (1992), Pagano (1993), Levine (1997, 1998), Levine et al. (2000), and Beck et al. (2000).

[10] Analysing the manufacturing sector in developing countries, Tybout (2000) illustrates that the emphasis on small scale production correlates negatively with the level of development

understand cross-country differences in TFP.

This is related to the third empirical fact we want to quantify, namely the positive effect of financial development on total factor productivity. Cross-country studies on the impact of credit market development on productivity are abundant. Beck, Levine and Loayza (2000) and Levine and Zervos (1998), among others, show that financial development positively and significantly affects total factor productivity[11].

While the link between financial frictions and misallocation has been studied before (e.g. Amaral and Quintin 2010 or Buera, Kaboski and Shin 2011), the focus of this paper will be on how the process of technology adoption and the learning-by-doing can be affected by financial frictions. More precisely, our theory builds on the previous chapter to develop an exogenous growth model where firms have different degree of access to the most advanced technology. The financial market is affected by the same two frictions from previous chapter, namely the imperfect enforcement and the asymmetric information. However, in this paper there is technological change and technology adoption with learning-by-doing. The frontier technology (the most advanced technology) grows exogenously. As the technology gradually spreads across firms, the learning-by-doing effect allows for better use of available technology. The technological experience (the weighted average of technology used in the past in the economy) affects also productivity.

There are two types of firms: Innovative, which have access to the frontier technology and improve technological experience (via the learning by doing process) and Backward which do not have access to frontier technology and do not contribute to the learning by doing process. Due to adverse selection, a higher contract enforcement increases resources allocated to the innovative firms and this will have two effects on productivity: 1) a static effect: they are more productive (as in the previous chapters) and 2) a dynamic effect: they have more weight and this increases the speed of learning by doing process, reducing the technological gap (with respect to the frontier technology).

Therefore, in our economy, the degree of enforcement will determine the optimal way to provide incentives (via the contracting environment) and, at the same time, will influence the efficiency of using available technologies (via the learning by doing process). We can then quantify how an exogenous variation in the capacity to enforce contracts affects resource allocation and the resulting output and productivity. To measure the importance of these effects, we calibrate our model to reproduce relevant features of the U.S. economy: We calibrate the productivity distribution for available technologies to fit salient features of the firm size distribution in the United States. Then we use the calibrated model to compare the predicted output and productivity with data available for a sample of 30 middle and high income countries, where the degree

[11] Hartmann et al. (2007) show that financial development in European countries has increased economy-wide productivity. Fisman and Love (2004) find that financial development promotes growth by allocating funds towards the most profitable investments. Arizala, Cavallo and Galindo (2009) find a significant impact of financial development on industry-level total factor productivity.

of contract enforcement is chosen such that our model generates average firm size of the magnitude observed in the data.

Amaral and Quintin (2010) and Buera, Kaboski and Shin (2011) also quantify the importance of limited enforcement and its effects on financial intermediation in the context of a dynamic general equilibrium model. However, the novelty of our approach resides in measuring the effects of limited enforcement in a framework which explicitly models adverse selection and the learning by doing process. Both of these channels have been mentioned in the theoretical literature as important for the process of financial and economic development (for a theory of enforcement with adverse selection see Erosa and Hidalgo 2008).

Firm size is affected in several ways. On the one hand, there is the static effect due to imperfect enforcement and asymmetric information. On the other hand, there is also the dynamic effect due to the technological gap between innovative and backward firms. As a consequence, the effect of financial frictions on firm size is amplified.

The dynamic effect of technological adoption in the presence of learning-by-doing also improves the prediction power of the model with respect to aggregate productivity in the economy. Our experiments indicate the technology adoption channel to be quantitatively important.

The paper is organized as follows. Section 2 presents the model economy. Section 3 characterizes agent's decisions, section 4 the optimal financial contract (developed in the first chapter) and section 5 the equilibrium and balanced growth path. Then, section 6 introduces the numerical experiments, while section 7 concludes.

2 The economy

This is an infinite horizon economy where the time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. There are three types of agents: households, firms and financial intermediaries. There is one final good that may be used for consumption or for investment.

2.1 Technology

There is a continuous technology indexed by its advancement level Z. At each period t, this technology can take values in the interval $[0, A_t]$, where A_t is the technological frontier, or the most advanced technology available at period t. We assume that the technological frontier evolves exogenously and grows at constant rate:

$$A_{t+1} = \Gamma A_t$$

where $\Gamma > 1$. We call technological experience $H_t \in [0, A_t)$ a weighted average of the technology used by workers in the past, where the previous period technology's weight is η:

$$H_{t+1} = \eta \int_0^{A_t} Z \, d\mu_t(Z) + (1-\eta) H_t = \eta \sum_{n=0}^{\infty} (1-\eta)^n \left[\int_0^{A_{t-n}} Z \, d\mu_{t-n}(Z) \right] \quad (1)$$

where $\mu_t(Z)$ is the measure of workers that work with technology Z at time t.

The technology Z is represented by a production function at firm level which uses three factors: firm's capital k_j, labor $l_{w,j}$ and managerial time $l_{m,j}$, where j is the firm index:

$$f(k_j, l_{w,j}, l_{m,j}; Z, H, A, \theta) = \begin{cases} 0 & \text{if } l_{m,j} \leq \varepsilon \\ Z\Psi(Z, H, A, \theta) \, (k_j)^{\alpha} \, l_{w,j}^{\beta} & \text{if } l_{m,j} \geq \varepsilon \end{cases}$$

where $\Psi(Z, H, A; \theta)$ is the state of know-how, and $\theta \in \{0, 1\}$ is a firm specific shock. A minimum level ε of managerial inputs is needed for the capital and labor to produce output. For the maximization problem of the firm to be well defined, we assume that $\alpha + \beta \in (0, 1)$. This production function allows us to obtain an endogenous firm size. This type of production function has been used, among others, by Antunes and Cavalcanti (2007), Guner, Ventura and Xu (2007), Perera-Tallo (2003, 2011), Amaral and Quintin (2010).

We assume the state of know how takes the form:

$$\Psi(Z, H, A; \theta) = \begin{cases} e^{-\frac{Z - [(1-\theta)H + \theta A]}{(1-\theta)H + \theta A}} & \text{if } Z \geq (1-\theta)H + \theta A \\ 1 & \text{otherwise} \end{cases}$$

The state of know how is a decreasing function of the advancement level of the technology Z and an increasing function of the technological experience H and the technological frontier A. Since the technological experience H is always smaller than the technological frontier A, the shock θ has a positive effect on the state of know how.

Both the technological experience H and the technological frontier A may be considered stocks of knowledge. The first stock of knowledge H represents the ability of using advanced technologies due to "learning by doing": the advancement level of the technologies used by workers in the past increases the ability of workers to use more advanced technology in the present (for other models of learning-by-doing see among others Arrow 1962, Krugman 1988; Stokey 1988; Lucas 1988, 1993; Matsuyama 1992; Young 1991; Perera-Tallo 2011).The second stock of knowledge A represents the "state of the art" in the most advanced scientific knowledge. Both stocks of knowledge have positive effect on the ability of workers to use efficiently advanced technology. That is, both of them rise the state of know-how.

The larger θ, the larger the positive effect that the technological frontier A will have on the productivity of the firm. Thus, the shock θ may be interpreted as the degree in which the firm has access to information about the most advanced technology. In order to simplify, we will consider that firms either have full access to the technological frontier, that is $\theta = 1$, or they do not have any access at all, $\theta = 0$. The probability that the firm has full access to the technological frontier ($\theta = 1$) is denoted by ν, hence the probability of not acceding at all to the technological frontier ($\theta = 0$) is $1 - \nu$.

Firms are perfectly competitive, there is free entry.

2.2 Households

In our model there are many identical households, each with a continuum of agents of measure one. The household values streams of the final consumption good according to the utility function:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(c_t) \tag{2}$$

where $\rho \in (0, 1)$ is the discount rate of the household, c_t is the average consumption of the household's members and the utility function is the CES utility:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \in (0,1) \cup (1, +\infty) \\ \ln c & \text{if } \sigma = 1 \end{cases}$$

The continuum of members in each household is introduced in the model in order to simplify, since this feature implies that households can perfectly diversify risk. As we will show, this assumption implies that we can represent the households behavior as the solution of a conventional representative household maximization problem with infinite horizon in a deterministic environment, and we can focus the analysis in the asymmetric information problem of firms and the learnig by doing effects that are really the contribution of our paper.

2.3 Financial intermediaries and the level of contract enforcement

Firms finance their production by signing contracts with financial intermediaries. These contracts are affected by two imperfections. On the one hand, the contracts are not fully enforceable. On the other hand, financial intermediaries cannot observe the idiosyncratic technological shock of the firms, therefore they don't know which firms have access to the frontier technology and which firms don't. Because contracts are not fully enforceable, firms can default. By defaulting, we mean that firms refuse to return the whole loan. When firm j defaults, the financial intermediary can recover the capital borrowed to the firm k_j and with probability λ can take also the cash flow of the firm, defined as output minus the payment to the workers: $\pi_j(k_j) = y_j - wl_{w,j}$. After the payment to the financial intermediary, managers are paid. It follows from the firm zero profit condition that managers receive the cash flow of the firms minus the payment to the financial intermediary.

Therefore, in case the firm defaults, the financial intermediary can only enforce the contract with probability $\lambda \in [0, 1]$, which is an index of the quality of the property rights. In this paper we will compare economies which differ only in this parameter λ of contract enforcement.

3 Agent decisions

3.1 Firms decision

At the beginning of period t, firms hire the managers, before the realization of the technological shock. This means that the number of managers cannot be contingent on the realization of the technological shock. Since the productivity shock is at this stage not known, the firm will specify the payment to the manager contingent on the productivity shock. Then the technological shock θ_j is realized. In order to produce firms need capital k_{jt}, which is financed through a financial intermediary, according to a contract we will analyze in the next section. After borrowing the capital, firms hire labor $l_{w,j}$, and use it together with the capital to produce the unique final good. After they produce, they sell the output to the households and pay the workers the wage w for their labor services.

Thus, firms chose the technology Z and the labor l_w once that they have chosen the amount of capital. This means that the maximization problem of the firm after they chose the capital is as follows:

$$\max_{l_w, Z\in[0,A]} Ze^{-\frac{Z-[(1-\theta_j)H+\theta_j A]}{(1-\theta_j)H+\theta_j A}} (k_j)^\alpha l_w^\beta - wl_w$$

First order conditions with respect to Z_j and l_w:

$$\begin{aligned}
\left(e^{-\frac{Z-[(1-\theta_j)H+\theta_j A]}{(1-\theta_j)H+\theta_j A}} - Ze^{-\frac{Z-[(1-\theta_j)H+\theta_j A]}{(1-\theta_j)H+\theta_j A}} \frac{1}{(1-\theta_j)H+\theta_j A}\right)(k_j)^\alpha l_w^\beta &= 0 \Leftrightarrow \\
Z &= (1-\theta_j)H+\theta_j A \qquad (3)\\
\beta Ze^{-\frac{Z-[(1-\theta_j)H+\theta_j A]}{(1-\theta_j)H+\theta_j A}} (k_j)^\alpha l_w^{\beta-1} &= w \Leftrightarrow \\
l_w &= \left(\frac{\beta Ze^{-\frac{Z-[(1-\theta_j)H+\theta_j A]}{(1-\theta_j)H+\theta_j A}} (k_j)^\alpha}{w}\right)^{\frac{1}{1-\beta}} = \left(\frac{\beta Z (k_j)^\alpha}{w}\right)^{\frac{1}{1-\beta}} \qquad (4)
\end{aligned}$$

Thus, the technological choice by the firms is a function of the shock θ_j:

$$Z(\theta_j) = \begin{cases} A & \text{if } \theta_j = 1 \\ H & \text{if } \theta_j = 0 \end{cases}$$

The firms that adopt more advanced technologies (A) are the one that have full access to the technological frontier ($\theta_j = 1$), and this is why we will call them from now on innovative firms and we will index them by i. The firms that do not have access to the technological frontier ($\theta_j = 1$) use a technology which is behind the most advanced one, and we will call them backward firms and we will index by b.

Notice that if we substitute the first order condition of the technology in the production function yields:

$$y_j = [(1-\theta_j)H + \theta_j A]\,(k_j)^{\alpha}\, l_w^{\beta} = \left[\left(\frac{\beta}{w}\right)^{\beta} [(1-\theta_j)H + \theta_j A]\,(k_j)^{\alpha}\right]^{\frac{1}{1-\beta}}$$

Therefore, the production of innovative and standard firms are respectively as follows

$$\begin{aligned} y_i &= A\,(k_i)^{\alpha}\, l_w^{\beta} = \left[\left(\frac{\beta}{w}\right)^{\beta} A\,(k_i)^{\alpha}\right]^{\frac{1}{1-\beta}} \\ y_b &= H\,(k_b)^{\alpha}\, l_w^{\beta} = \left[\left(\frac{\beta}{w}\right)^{\beta} H\,(k_b)^{\alpha}\right]^{\frac{1}{1-\beta}} \end{aligned}$$

Using equation (3) and (4), it follows that the cash flow of firms is as follows:

$$\pi_j(k_j) = (1-\beta)\left[\left(\frac{\beta}{w}\right)^{\beta} [(1-\theta_j)H + \theta_j A]\; k_j^{\alpha}\right]^{\frac{1}{1-\beta}}$$

That is:

$$\begin{aligned} \pi_i(k_i) &= (1-\beta)\left[\left(\frac{\beta}{w}\right)^{\beta} A\, k_i^{\alpha}\right]^{\frac{1}{1-\beta}} \\ \pi_b(k_b) &= (1-\beta)\left[\left(\frac{\beta}{w}\right)^{\beta} H\, k_b^{\alpha}\right]^{\frac{1}{1-\beta}} \end{aligned}$$

The decision about the amount of capital that firms get will be analyzed in section 4, where the optimal contract between firms and financial intermediaries is examined.

3.2 Households decision:

Every period t households are endowed with one unit of labor. At the beginning of each period, each member of the household makes an occupational choice between being a worker or a manager. If she decides to be a worker, she will earn a wage w_t. If she decides to be a manager, she will earn a payment contingent on the technological shock of the firm: she will earn w_{it}^{M} when the firm she manages is innovative (it receives the shock $\theta_j = 1$, which means that the firm has access to the technological frontier) and w_{bt}^{M} when the firm is backward (it receives the shock $\theta_j = 0$, which means that the firm does not have access to the frontier technology).

The household chooses at every period t the per capita consumption c_t and the share of household members who become workers denoted l_t, where $l_t \in (0, 1)$. The household's per capita assets at each period a_t are deposited with financial intermediaries which pay an interest r_t on them. The household takes as given the prices $\{r_t, w_t, w_t^M\}_{t=0,\infty}$ when making its decision on $\{c_t, l_t,\}_{t=0}^{+\infty}$.

The maximization problem of the households is as follows:

$$\begin{aligned} \max_{\{c_t,l_t,\}_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t u(c_t) \\ c_t + a_{t+1} - a_t &= w_t l_t + (1-l_t)w_t^M + r_t a_t \end{aligned} \tag{5}$$

where $w_t^M = \nu\, w_{it}^M + (1-\nu)w_{bt}^M$ is the average wage of managers. Notice that, since there is a continuum of members in each household, the household can perfectly diversify the risk associated to the managers' payment. This is why, when choosing how many households members are devoted to be worker or to be manager, the relevant variables are the wage and the expected payment of managers. The first order conditions of the household problem are:

$$u'(c_{t+1})\frac{1+r_{t+1}}{1+\rho} = u'(c_t) \tag{6}$$

$$w_t = w_t^M \tag{7}$$

where the first equation is the Euler Equation, while the second is an arbitrage condition: the wage of the workers w_t should be equal to the expected payment to managers w_t^M. Since, due to the continuum of members, the households can perfectly diversify the risk of the manager payment, the wage of workers is equal to the expected payment of managers.

4 The optimal financial contract

In this section we analyze the optimal financial contract. In order to simplify the exposition, along this section we will refer to the profits of firm before paying the managers simply as profits: $\pi_j(k_j) - (r_j + \delta)k_j$. Along this section we will consider both wages and deposit interest rates as given.

We've seen that there are two types of firms operating: innovative (highly productive firms) and backward (less productive firms). The financial intermediaries will offer, for each of the two types of firms, a contract consisting in an amount of capital k_j and an interest rate r_j. Given that financial intermediaries act perfectly competitive, expected (average) profits of financial intermediaries should be zero. In order to define the expected profit of financial intermediary, we will use the indicator function $\chi_j(r_j, k_j; \lambda)$ to denote the range of interest rate and capital at which the firm of type j will not default:

$$\chi_j(r_j, k_j; \lambda) = \begin{cases} 1 & \text{if } \pi_j(k_j) - (1+r_j)k_j \ \geq \ (1-\lambda)\pi_j(k_j) \\ 0 & \text{if } \pi_j(k_j) - (1+r_j)k_j \ < \ (1-\lambda)\pi_j(k_j) \end{cases}$$

The above indicator function means that if $\chi_j = 1$, the firm fulfills the contract, while if $\chi_j = 0$, the firm defaults. Thus, the expected profit by financial intermediaries is as follows:

$$\begin{aligned} &\nu\left[\chi_i(1{+}r_i)k_i + (1\text{-}\chi_i)\pi_i(k_i)\text{-}(1{+}r)k_i\right] + \\ &(1\text{-}\nu)\left[\chi_b(1{+}r_b)k_b + (1\text{-}\chi_b)\pi_b(k_b)\text{-}(1{+}r)k_b\right] \ = \ 0 \end{aligned} \tag{8}$$

The revenue of a financial intermediary from a contract is equal to $(1{+}r_j)k_j$ when the firm fulfills the contract $(\chi_j = 1)$ and $\lambda\pi_j(k_j)$ when the firm does not fulfill the contract $(\chi_j = 0)$, while the cost of a contract in terms of the payment to depositors is $(1{+}r)k_j$.

This condition will be referred as financial intermediaries zero profit condition.

At equilibrium it should not be possible for a financial intermediary to make a better offer to any type j firm, without incurring a loss. Due to the two imperfections in the functioning of the financial markets (imperfect enforcement and asymmetric information), the menu of contracts will maximize profits for each firm type, taking into account two incentive constraints: *i*) first, firms should have incentives to repay their loans, *ii*) second, firms should have incentive to truly reveal their type.

The first constraint refers to the incentive for firms to repay the financial intermediary and we will call it the non-default constraint:

$$\pi_j(k_j) - (r_j+\delta)k_j \ \geq \ (1-\lambda)\pi_j(k_j), j \in \{i, b\} \tag{9}$$

If a firm defaults, the financial intermediary gets her capital after depreciation $(1{-}\delta)k_j$, and with probability λ he also gets her cash flow, while the firm gets the expected amount $(1-\lambda)\pi_j(k_j)$. If the firm fulfills the contract she gets her cash flow minus the payment to the financial intermediary: $\pi_j(k_j) - (r_j+\delta)k_j$. This incentive constraint is saying that the firm is better off paying to the financial intermediary, in which case she gets $\pi_j(k_j) - (r_j+\delta)k_j$ than defaulting, in which case she gets $(1-\lambda)\pi_j(k_j)$.

The second constraint refers to the incentive for each firm to correctly reveal her type, and we will call it revelation constraint:

$$\pi_b(k_b) - (r_b+\delta)k_b \ \geq \ \max\{(1-\lambda)\pi_b(k_i), \pi_b(k_i) - (r_i+\delta)k_i\} \tag{10}$$

$$\pi_i(k_i) - (r_i+\delta)k_i \ \geq \ \max\{(1-\lambda)\pi_i(k_b), \pi_i(k_b) - (r_b+\delta)k_b\} \tag{11}$$

This incentive constraint says that the backward type firm should be better off to sign the contract destined to her own type, getting $\pi_b(k_b) - (r+\delta)k_b$ as a payoff, rather than pretending to be an innovative type, getting the payoff $\max\{(1-\lambda)\pi_b(k_i), \pi_b(k_i) - (r_i+\delta)k_i\}$. The same should be true for the innovative type firm.

4.1 The equilibrium menu of contracts

Definition 1 *A menu of contracts* $\{(r_i, k_i), (r_b, k_b)\}$ *satisfying the financial intermediary zero profit condition (8) and the revelation constraints (10,11) represents an equilibrium menu of contracts if*

1. $\forall j \in \{i, b\}$ there is no other contract (r'_j, k'_j) for the type j in which the firm j is better off $\theta_j \pi_j(k'_j) - (1 + r'_j)k'_j > \theta_j \pi_j(k_j) - (1 + r_j)k_j$ and in which financial intermediary gets at least zero profit $\chi_j(1+r_j)k_j + (1-\chi_j)\lambda\theta_j\pi_j(k_j) \geq (1+r)k_j$ and in which the revelation constraints (10,11) hold.

2. There is no other menu of contracts $\{(r'_i, k'_i), (r'_b, k'_b)\}$, in which financial intermediary gets at least zero profit (8) and in which $\theta_j \pi_j(k'_j) - (r'_j + 1)k'_j \geq \theta_j \pi_j(k_j) - (1 + r_j)k_j \ \forall j \in \{i, b\}$, being the last inequality strict for one of the types, and in which the revelation constraints (10,11) hold.

Thus, a menu of contracts is an equilibrium if it satisfies the financial intermediary zero profit condition (8) and the revelation constraints (10, 11), and it is not possible to find, for one or both firm types, a better contract where non-negative financial intermediary profit condition is satisfied. It is still possible a different contract in which both types have the same contract and in which the backward type defaults: it is the pooling contract. Nevertheless, it is well known that this type of pooling contract is never an equilibrium. Furthermore, Chapter 1 shows that if there are enforcement cost (or state verification cost) in case of default, and these costs are high enough, the pooling contract is never superior to the separating menu of contract.

We note that an equilibrium menu of contracts will have $r_b = r_i = r$. Otherwise, the financial intermediary would be making positive profits with the contract of at least one firm type. This couldn't be sustained as an equilibrium because it would always be possible to offer a better contract to this type.

As the innovative type firm has higher productivity, she gets a larger amount of capital than the backward type firm[12]. This means that the innovative type firm has never the incentive to pretend to be a backward type. Only the backward type firm has incentives not to correctly reveal her type. This means that the revelation constraint (11) is never binding and consequently is irrelevant for the subsequent analysis.

It is possible to prove that the revelation constraint (10) is always tighter than the non-default constraint for the innovative type. Thus, the only incentive constraint relevant for the innovative type is the revelation constraint of the backward type.

To sum up, if $\{(r_i, k_i), (r_b, k_b)\}$ is a menu of contracts, then $r_i = r_b = r$ and:

[12] Chapter 1 offers a formal proof of the statements in this section.

$$
\begin{aligned}
k_b = \quad & \arg\max_k \pi_b\left(k\right) - (r+\delta)k \\
& \qquad s.t.\ \pi_b\left(k\right) - (r+\delta)k \ \geq \ (1-\lambda)\pi_b\left(k\right)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
k_i = \quad & \arg\max_k \pi_i\left(k\right) - (r+\delta)k \\
& \qquad s.t.\ \pi_b\left(k_b\right) - (r+\delta)k_b \geq (1-\lambda)\pi_b\left(k_i\right)
\end{aligned} \tag{13}
$$

Thus, the menu of contracts is characterized by the following features: $i)$ the interest rate that financial intermediaries charge to firms is the depositors interest rate; $ii)$ there is no default in equilibrium since non default constraint is satisfied, this and the above features guarantee the zero profits of financial intermediaries; $iii)$ The only incentive constraint that is relevant for the backward type is the non default constraint, since the innovative type never has incentives to cloak his type. The only incentive constraint that the innovative type faces is the revelation constraint, which is always tighter than the non-default constraint for the innovative type. $iv)$ The menu of contracts is such that firms of each type maximize profits subject to the relevant incentive constraint for each type (the non-default constraint for the backward type and the revelation constraint for the innovative type).

The upper part of Figure 1 displays the non default constraint (9) for the bad type: the non default constraint is satisfied when the profit in case of fulfilling the contract, which is hump-shaped, is superior to the profit in case of default, which is an increasing curve. In the lower part of Figure 1 is represented another version of the non default constraint: the bad type firm has incentive to fulfill the contract if the expected average payment to the financial intermediary in case of default, which is a decreasing function in k_j, is higher than in case of fulfilling the contract, which is constant. It is apparent from lower part of Figure 1 that the non-default constraint holds when the amount of capital is smaller than a certain threshold capital in which the non-default constraint holds with equality. Such level of capital is the maximum amount that the bad firm is going to be able to borrow. This will be called the non default borrowing limit, and we will denote it by k_b^{nd}:

$$
k_b^{nd} \Leftrightarrow \pi_b\left(k_b^{nd}\right) - (r+\delta)k_b^{nd} \ = \ (1-\lambda)\pi_b\left(k_b^{nd}\right) \ \text{(and } k_b^{nd} > 0\text{)}
$$

Figure 2 displays the revelation constraint which will affect the innovative type: the profits of the backward firm fulfilling her contract should be larger than the profits of the backward firm when she accepts the contract of the innovative type and defaults. The cash flow function is strictly increasing, which implies that the profits of the backward firm in case of default are increasing. It follows that the revelation constraint holds when the amount of capital is smaller that certain threshold capital for which the revelation constraint holds with equality. Such level of capital is the maximum amount that the innovative firm is going to be able to borrow, and it will be called revelation borrowing limit, denoted by k_i^{rv}:

$$
k_i^{rv} \Leftrightarrow \pi_b\left(k_b\right) - (r+\delta)k_b \ = \ (1-\lambda)\pi_b\left(k_i^{rv}\right) \ \text{(and } k_i^{rv} > 0\text{)}
$$

Thus, another way to write the menu of contracts is as follows:

$$k_b = \underset{k}{\arg\max} \pi_b(k) - (r+\delta)k \quad s.t.\ k \le k_b^{nd} \tag{14}$$

$$k_i = \underset{k}{\arg\max} \pi_i(k) - (r+\delta)k \quad s.t.\ k \le k_i^{rv} \tag{15}$$

4.2 Financial contract and the degree of contract enforcement

In this subsection we characterize the optimal financial contract that financial intermediary offers in equilibrium to each firm type (backward and innovative), as a function of the degree of contract enforcement.

Considering the backward firm, when the contract enforcement goes down, the punishment for default falls as well. This implies that the maximum amount of capital that the backward firm can receive - the non default borrowing limit k_b^{nd} - decreases. Thus, after a drop in contract enforcement, it is more likely that the firm is constraint, or in order words that the non-default constraint is binding. If the firm was already constrained, the fall in the contract enforcement reduces the non default borrowing limit and consequently the amount borrowed by the backward firm, lowering her profits.

But the fall of the contract enforcement also affects the innovative firm. As we have seen, the fall in the contract enforcement may reduce the profits of the backward firm (left hand side of the revelation constraint 13), which reduces the "positive incentives" to reveal her type. Furthermore, a drop in the contract enforcement also reduces the punishment when the backward firm does not reveal her type (see right hand side of the revelation constraint 13). Thus, the fall in the contract enforcement reduces also the revelation borrowing limit k_i^{rv}, which affects the innovative firms, making more likely that such firms are constrained and reducing the capital received by them in case that these firms are already constrained.

Summarizing, the fall of the contract enforcement will reduce both the non default borrowing limit of the backward firms and the revelation borrowing limit which affects innovative firms, making more likely that firms are constrained and reducing the amount of capital that firms borrow in case that these firms are already constrained. Thus, a fall of the contract enforcement tights the incentive constraints and affects firm size distribution by making firms smaller than their optimal size, which in turn it reduces the productivity of firms.

The unconstrained maximization of profits implies optimum amounts of capital for each of the two firm types, which we will denote in our analysis by k_j^* with $j \in \{i, b\}$:

$$k_j^* = \underset{k}{\arg\max} \pi_j(k) - (r+\delta)k \Leftrightarrow \frac{\partial \pi_j\left(k_j^*\right)}{\partial k} = (r+\delta)$$

It follows from the implicit function theorem that innovative firms receive more capital than backward ones (i.e. $k_i^* > k_b^*$). We will use this particular case a benchmark/starting point for our analysis.

The following proposition summarizes how the level of contract enforcement affects the allocation of capital for the two types of firms (proof in Chapter 2).

Proposition 1 *There are two threshold contract enforcement*[13] λ_1 *and* λ_2*, where* $1 > \lambda_2 > \lambda_1 > 0$ *and such that: i) If* $\lambda \geq \lambda_2$ *then neither the innovative or the backward firms are constrained:* $k_j = k_j^*$*. ii) If* $\lambda \in [\lambda_1, \lambda_2)$ *then innovative firms are constrained,* $k_i < k_i^*$*, while backward firms are not,* $k_b = k_b^*$*, the amount of capital that innovative firms receive* k_i *increases with* λ*. iii) If* $\lambda < \lambda_1$*, both types of firms are constrained and the amount of capital that each type receives is the same:* $k_i = k_b < k_b^*$*; furthermore, the amount of capital that firms receive increases with* λ*.*

Thus, when the level of contract enforcement drops, the incentives that firms have to fulfill financial contracts and to truly reveal their type fall as well. This makes incentive constraints tighter, reducing the non default borrowing limit, which affect backward firms, and the revelation borrowing limit, which affects innovative firms. There are three possible situations depending on the level of contract enforcement. When the contract enforcement is good enough but not necessary perfect, i.e. larger than λ_2, then incentive constraints are not binding and firms chose their optimal capital level. When the contract enforcement is in a middle range, $\lambda \in [\lambda_1, \lambda_2)$, then only innovative firms are constrained while backward firms are not. In this sense, innovative firms suffer from an incentive constraint (the revelation incentive constraint) that is tighter than the one that affects backward firms (the non default constraint). Finally, when the level of contract enforcement is poor, $\lambda < \lambda_1$, both firms are constrained and receive the same inefficiently small amount of capital. Furthermore, when firms are constrained and the level of contract enforcement improves, borrowing limit expand making firms closer to their optimal level and consequently more productive.

Thus, the link between property rights quality and productivity is clear from this proposition. Weak property rights make firms to have an inefficiently small size. Furthermore, weak property rights affect more innovative firms, and influence the way in which capital is distributed among firms, pouring relatively more resources in backward, less productive firms and less resources in the innovative, more productive firms. These two mechanisms, the reduction of firms size and the redistribution of resources from more productive to less productive firms, imply that the quality of property rights will affect positively the productivity. But there is still a third mechanism, which is of dynamic nature. A fall in the level of enforcement will reduce the amount of resources that innovative firms receive, consequently the spread of innovation, and the learning by doing process will slow down (see equation 1). As a consequence, the

[13] The exact definition of these thresholds are: $\lambda_1 \equiv \frac{\alpha}{1-\beta}$ and $\lambda_2 \equiv \frac{\alpha}{1-\beta} + \left[\left(1 - \left(\frac{H}{A}\right)^{\frac{\alpha}{(1-\beta)(1-\alpha-\beta)}}\right)\frac{1-\alpha-\beta}{1-\beta}\right] \Leftrightarrow \lambda_2 \equiv 1 - (\frac{H}{A})^{\frac{\alpha}{(1-\beta)(1-\alpha-\beta)}}(\frac{1-\alpha-\beta}{1-\beta})$

gap between the average technological level in the economy and the technological frontier will rise. This means that the productivity gap between countries with weak property rights and countries with strong property rights will exacerbate due to the slow technological spread in countries with weak property rights.

5 Equilibrium and Balanced Growth Path

Definition 2 *An equilibrium is an allocation* $\{c_t, a_t, l_t, n_t, \{k_{jt}, l_{wjt}, Z_{jt}\}_{j\in\{i,b\}}, H_t\}_{t=0}^{\infty}$, *where* n_t *is the number of firms, and a vector of prices* $\{r_t, w_t, \{r_{jt}, w_{jt}^M\}_{j\in\{i,b\}}\}_{t=0}^{\infty}$ *such that* $\forall t \in \{0, 1, 2, ...\}$*:*

1. Households choose $\{c_t, a_t, l_{t,}\}_{t=0}^{\infty}$ in order to maximize their utility (2) by taking as given the prices $\{r_t, w_t, w_t^M\}_{t=0,\infty}$ and choosing subject to their budget constraint (5).
2. Financial intermediaries offer to each firm of type $j \in \{i, b\}$ a contract $\{k_{jt}, r_{jt}\}_{j\in\{i,b\}}$ that is an equilibrium contract according to definition 1.
3. Firms choose how many workers $\{l_{wjt}\}_{j\in\{i,b\}}$ to hire and the technology $\{Z_{jt}\}_{j\in\{i,b\}}$ to be used in order to maximize their cash-flow, taking as given the wage w_t, and the amount of capital k_{jt} offered in the menu of contracts.
4. Free entry implies that firms get zero profits: $\pi_j(k_j) - (1+r_j)k_j - w_{jt}^M\varepsilon = 0$ $j \in \{i, b\}$.
5. Labor market clears:
- Workers Market: $n_t[\nu l_{wit} + (1-\nu)l_{wbt}] = l_t$.
- Manager Market: $n_t\varepsilon = 1 - l_t$
6. Capital market clears: $n_t[\nu k_{it} + (1-\nu)k_{bt}] = a_t$.
7. Technological experience follows the law of motion (1).

Thus, the definition of equilibrium is the usual one: agents maximize their objective function and markets clear.

Definition 3 *Balanced growth path is an equilibrium where* $\{c_t, a_t, w_t, \{k_{jt}, w_{jt}^M, Z_{jt}\}_{j\in\{i,b\}}, H_t\}_{t=0}^{\infty}$ *grow at the same rate* $(\Gamma - 1)$ *and* $\{l_t, n_t, r_t, \{r_{jt}, l_{wjt}\}_{j\in\{i,b\}}\}_{t=0,\infty}$ *are constant over time.*

6 Contrasting economies with different Contract Enforceability

We consider balanced growth path equilibria of economies that differ only in the property rights quality λ. We analyze how the output, the Total Factor Productivity and the average establishment size change when we move from a lower to a higher enforcement. We are particularly interested to what extent the mechanisms linking property rights quality to these variables are able to explain the variation of these variables observed in the data.

6.1 Parameterization

We followed the 'standard practice' in macro models whenever possible. When existing work provided less guidance, we mainly relied on the model's implications on the size distribution of firms. To follow the common practice in the literature, we chose the parameter values so that our model, with perfect contract enforcement , at the steady state, matched data at aggregate and cross section level for the United States economy. We took the United States economy as a good approximation, for our purposes, of a distortion-free economy (which in the context of our model translates into an economy with perfect contract enforcement. i.e. with the enforcement parameter λ equal to one).

Table 1 summarizes the parameters and the targets used for calibration.

Parameter	Meaning	Value	Target
α	capital share	0.36	Capital elasticity based on Cooley and Prescott (1995)
δ	depreciation rate	0.09	Ratio between investment and capital
ρ	discount rate	0.05	Capital-output ratio
$\Gamma - 1$	growth rate	0.016	Average growth rate of US output/worker (PWT 7.0)
ν	share of innovative firms	0.16	Innovative firms are the ones who hire more employees than the average no of employees (2002 US Ec. Census)
β	labor share	0.58	The share of labor and the share of output for innovative firms (2002 US Ec. Census)
ε	managerial inputs per establishment	3.9	Average no of employees for manufacturing establishments (2002 US Ec. Census)
$\frac{A}{H}$	gap b/w tech. frontier and tech. experience	1.24	The share of output corresponding to innovative firms
η	law of motion of technological change	0.08	Share of workers in innovative firms
σ	elasticity of inter-temporal substitution	1.7	

Table 1: Parameter values (source: authors calculations)

For the share of capital in total output, α, we applied the methodology described in Cooley and Prescott (1995) on data from the National Income and Product Accounts. The share of capital α averages 0.36 for the period 1950-2010. We compute, using the same data source, the depreciation rate δ as the ratio between investment and capital and we obtain a value of 0.09.

For the discount rate, assuming CES utility function, it follows from the Euler Equation that:

$$\frac{c_{t+1}}{c_t} = (1 + \upsilon_{USA}) = \left(\frac{1+r}{1+\rho}\right)^{1/\sigma}$$

where υ denotes the growth rate of per capita GDP that in our model, along the balanced growth path, is equal to the consumption growth rate. We compute the net interest rate as the net interest rate, that in our model (in the benchmark case) is equal to the net marginal productivity of capital, which is as following:

$$r = \alpha \frac{Y}{K} - \delta$$

Hence $(1 + \upsilon_{USA}) = \left(\frac{1-\delta+\frac{\alpha y}{k}}{1+\rho}\right)^{1/\sigma} \Leftrightarrow \rho = \frac{1-\delta+\frac{\alpha y}{k}}{(1+\upsilon_{USA})^{\sigma}} - 1.$

For the capital-output ratio, we used data from the Bureau of Economic Analysis for the period 1950-2010. We defined the stock of capital as being comprised by private fixed capital (nonresidential plus residential), durable goods and inventories. The definition output Y is the same we use when computing the capital share α. The result was a capital-output ratio of 2.42 and a discount rate of 0.05.

The growth rate of the technological frontier Γ is chosen to match the USA growth rate (see appendix for details):

$$\Gamma = (1 + \upsilon_{USA})^{1-\alpha}$$

where υ_{USA} = average growth rate of the USA output per worker, computed from the data compiled by PWT 7.0. The average growth rate for the real GDP per worker (series rgdpwok) for the period 1950-2010 implied a growth rate of the technological frontier of approximately 1.6% per year.

For the rest of parameters, we tried to match in the best possible way the distribution of firm size generated by our model with the distribution of firm size given by available data for the U.S. We used the 2002 U.S. Economic Census which provides information on number of employees, valued added and many other variables for the establishments grouped by the North American Industry Classification System (NAICS)[14]. Apendix **??** offers suplementary details on the calibration of these parameters.

First we need to decide which industry sectors from NAICS we want our model to describe. Our objective is to make cross-country comparisons regarding the importance of different property rights enforcement on output, firm size and TFP, therefore we would like to use data on firm size as comparable as possible across countries. Various industry sectors covered by NAICS vary as importance across economies; given the different optimal firm size across industry sectors, we focus on manufacturing. In this way, we try to minimize the effect of different industry structure across countries (which is not the focus in our model) and to "isolate" as much as possible the effect of property rights from other factors that may affect firm size.

In our model there are two types of firms, innovative (larger and more productive) and backward (smaller and less productive). The US Economic Census groups establishments in 10 size classes (starting with '1-4' employees up to 'more than 2500' employees, see Table 2). We note that the average value

[14] As a classification of economic activities used in North America, NAICS is based on the United Nations' Standard Industrial Classification (SIC), just as the Statistical Classification of Economic Activities in the European Community (NACE).

The data from the 2002 U.S. Economic Census that we used is freely available at the U.S. Census Bureau website.

added per employee is increasing with firm size. Hence, the Census data seems consistent with the model's implication that the larger firms are also the more productive ones.

We need to match the firm size classes reported in the Census to the two firm types in our model. We calculate the average across all the firms in the data set, and we get that this average is 41,9. Then, we take those size classes employing on average more workers than the sector average (of 41,9) to belong the "innovative" type (with larger and more productive firms). In this way we avoid imposing a definition of what is a 'large' firm. We obtain a share of innovative firms ν of 16% of the total number of firms.

The technological gap between the two firm types $\frac{A}{H}$ is obtained from matching the share of output corresponding to large firms. We note that once we set the ratio of A/H, we do not have to set a specific value for H. While affecting the level of capital and output in equilibrium, H does not affect the results we are interested in - the average establishment size and the relative output. For the parameter η from the law of motion of technological change, we note that at Steady State $\frac{A_t}{H_t}$ is constant, so $\frac{H_{t+1}}{H_t} = \Gamma$, and

$$\eta = \frac{\Gamma - 1}{\mu_t(A_t)\left(\frac{A_t}{H_t} - 1\right)}$$

The share of workers in high productivity firms is $0,8$, resulting in η of $0,08$.

We then chose β such that the benchmark model matches the share of labor and output corresponding to innovative (more productive) firms. In this way we obtained that the coefficient of labor (workers) in the production function β is equal to 0.58, which implies returns to scale $\alpha + \beta$ of 0.94. We note that this value is close to values used by Guner, Ventura and Xu (2006), Basu and Fernald (1997), Chang (1998), Antunes and Cavalcanti (2007).

The managerial inputs per establishment, ε, is such that the benchmark model matches the average establishment size, which for our data set is of 41,9 employees. We obtain a value of $3,9$ managers per firm. We note that this implies that smaller firms have a higher fraction of managerial inputs. It seems plausible that in small firms the effort input of the managers represents a more significant part of the total labor effort involved in the firm compared to large firms.

6.2 Results

In this subsection we will check how our model with endogenous technological adoption fares in explaining the differences across countries in income, productivity and firm size. We will consider economies on the balanced growth path, where capital and output grow at the same exogenous rate as the technological frontier $(\Gamma - 1)$. In our experiments, economies differ in one single aspect: the degree of contract enforcement, modelled by the share λ of cash flow that financial intermediaries can recover from the firms in case of default. In particular, one of the innovative features of our model was the explicit inclusion of technology accumulation in the spirit of the learning by doing literature. Naturally, we would like to check if this modelling approach makes a difference for the analysis. To this purpose, we will compare the model's prediction to the version of the model where technological change is exogenous, which was treated in Chapter 2.

Figures 3 and 4 plot the output and the total factor productivity against the degree of property rights enforcement (the continuous lines). The first observation is that the model is *qualitatively* consistent with the empirical correlation between enforcement, output and productivity. The mechanism affecting aggregate output and productivity works in our model as follows. As explained in section 4.2, when property rights are weak, firms have an inefficiently small size, which can be gauged also from the positive correlation between the contract enforcement and the average firm size, illustrated in Figure 5. Furthermore, weak property rights affect to a greater extent the innovative firms, influencing the way capital is distributed to firms: backward firms will receive relatively more resources, while innovative firms will receive less.

This will feed into a third mechanism, which is of dynamic nature and is driven by the process of accumulation of technological experience. The technological experience is the stock of knowledge representing the ability of using advanced technologies due to "learning by doing". Given that a fall in the level of enforcement reduces the amount of resources that innovative firms receive, the adoption of advanced technology and the learning by doing will slow down. Then, the average technology (employed by backward firms) will fall behind the technological frontier (employed by innovative firms), that is the gap between the two will rise. This will exacerbate adverse selection problems and will increase the productivity gap between countries with weak property rights and countries with strong property rights, as backward firms employ relatively more resources in the former than in the latter.

Therefore, although the technological frontier evolves exogenously, the accumulation of technological experience differs across economies with different property rights. More innovative firms receive relatively more capital (and consequently hire more workers) in countries with higher property rights enforcement. Given that more innovative firms use an advanced technology, the overall economy benefits from a higher stock of technological experience, which in turn contributes to increase the overall productivity as explained above.

The technological gap of backward firms relative to the frontier technology, computed as $\left(\frac{H}{A}\right)^{\frac{1}{1-\alpha}}$, is plotted in Figure 6 against the level of property rights enforcement. In an economy with perfect enforcement, the backward firms use a technology which is about 30% less productive than the frontier technology. This holds in the model where technology adoption is exogenous, treated in Chapter 2 (dotted line in Figure 6), and also in the model where technology adoption is endogenous (continous line in Figure 6). However, in the model where technology adoption is exogenous, the technological gap of backward firms does not change with enforcement, while in the model with endoenous technology adoption the technological gap does change: as illustrated in Figure 6, the technological gap increases when enforcement deteriorates, and at very low levels of enforcement the backward firms use a technology about 50% less productive than the frontier technology. Therefore, our theory with endogenous technological adoption implies that the differences in productivity among innovative and backwards firms are more significant for less developed countries, which is consistent with what Erosa and Hidalgo (2008) report on industries in the manufacturing sector.

The change in the technological gap of backward firms relative to the frontier technology amplifies the differences in output per capita generated by the model. The quantitative importance of this effect can be seen in Figure 3 where the prediction of the model with exogenous technology adoption from Chapter 2 (in dotted line), is plotted along the prediction of the current model. On average, the current model predicts an output about 25% smaller than the model with exogenous technology adoption.

Nontheless, our main interest in this Chapter has been to generate a higher variation in TFP. The predictions of the two models for TFP are compared in Figure 4. While the model with exogenous technology adoption generates a variation of TFP of 30%, the explanatory power of the current model is increased to 50%.

6.3 Results vs. cross-country data

To put the above results into context, we check how the current model fares in explaining the cross-country differences in output per worker and TFP observed in data. We decide to plot the output per worker and the TFP against the average firm size. Alternatively, they could be plotted against a measure of the quality of property rights λ. We chose to plot it against the average firm size because for the parameter interval we are performing the comparison the average firm size moves monotonically with the quality of property rights and it has a clear empirical counterpart[15].

Following Amaral and Quintin (2010), as a measure of the relative economic development across countries we used the output per worker. The data source is Penn World Table (PWT) version 7.1 and the variable is rgdpwok. The variable is intended to control for differences rates of employment and price levels across countries (which are not the focus of our analysis). Although the relative GDP per worker displays variation over time, our interest is on a long term average of the macro variables. We used the average for the period 1995 up to most recent data, for two reasons: on the one hand, for this period the series are more stable and at the same time, the closer we are to present, generally the more reliable can be considered the series.

Reliable and comparable data on firm size is scarce and, in general, available in countries with a sufficient statistical development that usually perform economic censuses regularly. We used data for 30 countries from the OECD Structural Business Statistics Database, whose level of development is from

[15] A discussion of this issue is done in Chapter 2, section 6.4.

'middle-income' until 'high income'. The relative GDP per worker with respect to the US value lies between 0.24 for Argentina up to 0.97 for Ireland. 13 countries in our sample have a GDP per worker below 0.6 of the US level, with the rest with values between 0.6 and 0.97[16]. We are missing in our sample the least developed countries, because data for these countries is hardly available and of lesser quality (the high level of non-declared activities in these countries makes it rather difficult to compile reliable statistics on firm size and GDP). The advantage of this data set is that is based on a common classification of activities (ISIC rev3). Given that the optimal scale of production varies across industries, we chose to focus on the manufacturing sector in our comparisons.

Figure 7 displays the relative GDP per worker predicted by our model versus the one in the data from PWT 7.1. For comparison, we plotted in the same graph the GDP per worker predicted by the two versions of the model (with and without endogenous technological adoption). The data in our sample confirms an empirical correlation between the quality of property rights and the average firm size, on the one hand, and the output per worker, on the other hand. Taking into account that we abstracted from several factors relevant for development (like human capital, monopoly arrangements, etc), both versions of our model seem to explain a large part of the variation in ouput per worker (with the model with endogenous technology adoption doing slighly better).

However, the main reason we considered the endogenous technology adoption was to improve the explanatory power for TFP. We compute TFP, as in the rest of this paper, by the following equation:

$$TFP = \left(\frac{y}{k^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

We estimate the stock of capital using the perpetual inventory method, where the depreciation rate is the one in our model and we consider the initial capital as the one predicted by the neoclassical model at the steady state. That is:

$$\begin{aligned} k_{\text{intial period}} &= \left(\frac{\alpha}{\delta+\rho}\right) y_{\text{intial period}} \\ k_{t+1} &= i_t + (1-\delta)k_t \end{aligned}$$

where we use as the initial period the first year available in the PWT for each country. Figure 8 presents the model predictions vs. the data. The differences in expanatory power of the two models are more important in the case of TFP than in case of ouput: the model with endogenous technology adoption clearly outperforms the previous version, as it is apparent also from the linear tendency lines.

[16]The World Bank maintains a classification of countries by income. The sorting for the countries in our sample, although not based on their classification, is broadly in line with it.

7 Conclusion

In this paper we investigated some mechanisms that may contribute to a better understanding of the observed differences across countries in output per capita, productivity and firm size. Several papers have shown that the quality of property rights can explain a significant part of the variation in output (Amaral and Quintin 2010, Buera, Kaboski and Shin (2011), Chapter 2 of this thesis).

Although these papers are able to generate important differences in output, the generated differences in TFP are relatively smaller. Given that TFP is usually interpreted as an index of technological advancement, in this paper we asked the question: what stops developing countries to adopt more advanced technologies from developed countries? The literature has shown that weak property rights can cause a bad allocation of funds to firms, where better (and more innovative) firms receive inefficiently low amounts of credit. We inquired whether this ineffective allocation of funds acted as a barrier to the spread of new technologies and we modelled this by introducing a learning-by-doing process, where the advancement level of the technologies used by firms in the past increases their ability to use more advanced technology in the present.

We investigated quantitatively this hypothesis and it turned out it has non-trivial consequences for the technological gap between backward and advanced firms in the economy. In our model, this technological gap is not constant any-more, but varies with the quality of property rights. Thus, a certain difference in property rights protection will feature an increased technological gap between backward and advanced firms, which exacerbates the asymmetric information problems. In turn, this amplifies the total effect of property rights on the aggregate productivity, the experiments indicating the channel to be quantitatively important.

One way our model could be extended would be to look at additional implications of property rights protection outside the financial intermediation process. For instance, the problems companies face in enforcing contracts with suppliers, but especially with clients could act as a break for expanding production and innovation processes. Modelling this from a macroeconomic general equilibrium perspective may be challenging but useful.

At the same time, a closer look at the differences in the economic structure and the firm size distribution among developing and industrialized economies may bring new insights in the understanding of the economic development of nations.

Size class	Number of establishments	Number of employees	Average number of employees per establishment	Value Added	Average Value Added per employee
All	350.828	14.699.536	41,9	1.887,7	128,4
1_4	141.992	279.481	1,97	21,6	77,3
5_9	49.284	334.459	6,79	28,3	84,6
10_19	50.824	702.428	13,82	58,0	82,6
20_49	51.660	1.615.349	31,27	142,8	88,4
50_99	25.883	1.814.999	70,12	181,7	100,1
100-249	20.346	3.133.384	154,00	357,4	114,1
250_499	6.853	2.357.917	344,07	297,0	126,0
500_999	2.720	1.835.386	674,77	286,1	155,9
1000_2499	1.025	1.494.936	1458,47	262,0	175,3
2500_	241	1.131.197	4693,76	252,4	223,1

Table 2: Descriptive statistics for the establishments in U.S. manufacturing sector (source: the 2002 U.S. Economic Census)

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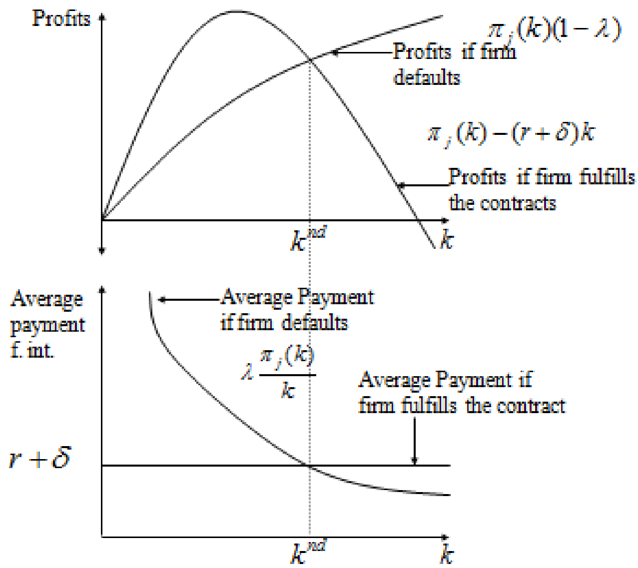

Figure 1: The Non-Default Constraint

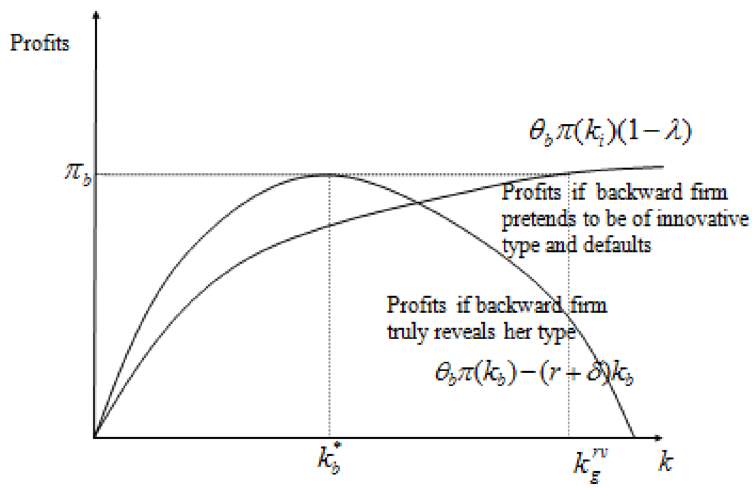

Figure 2: The Revelation Constraint

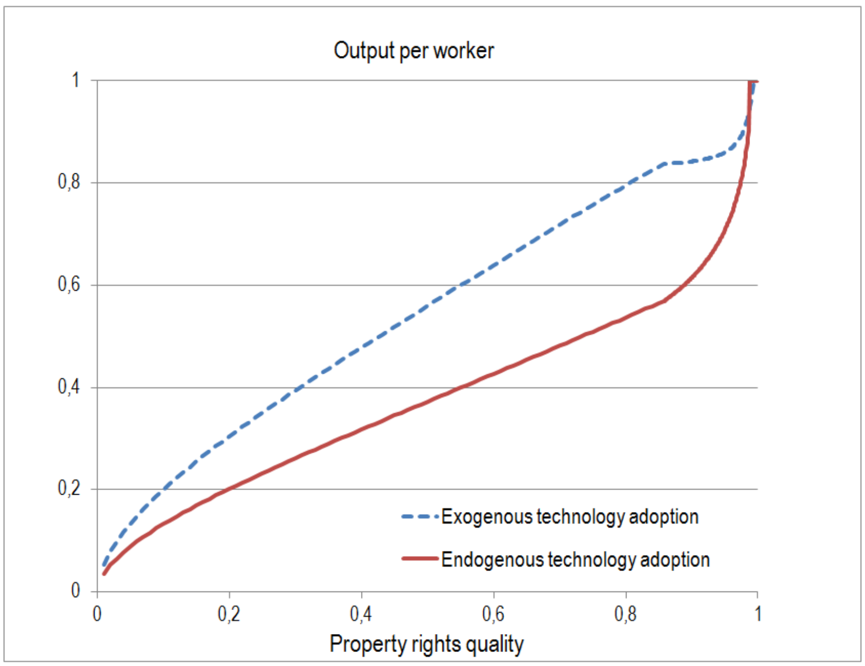

Figure 3: Impact on output.

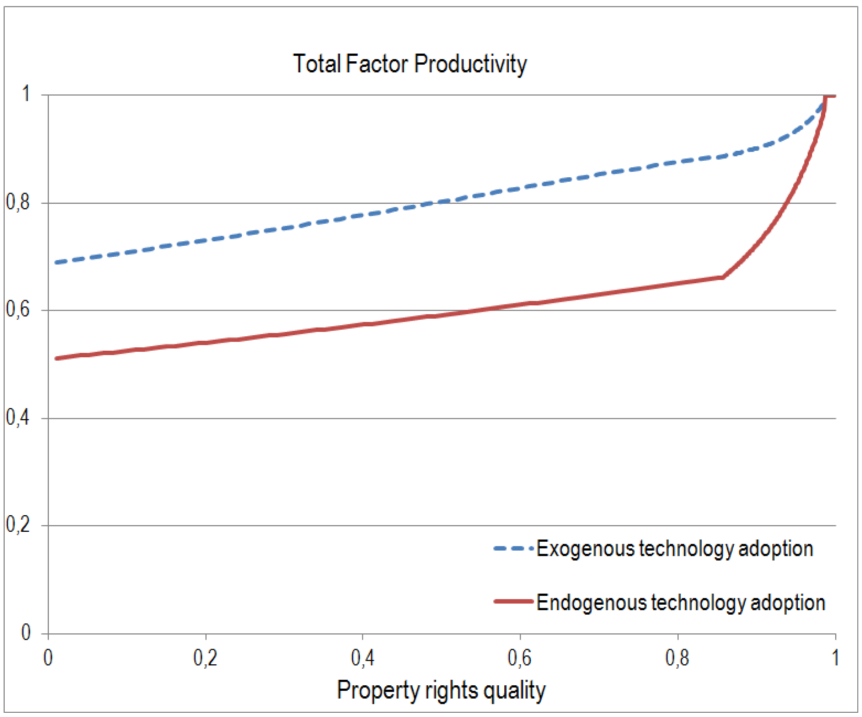

Figure 4: Impact on TFP

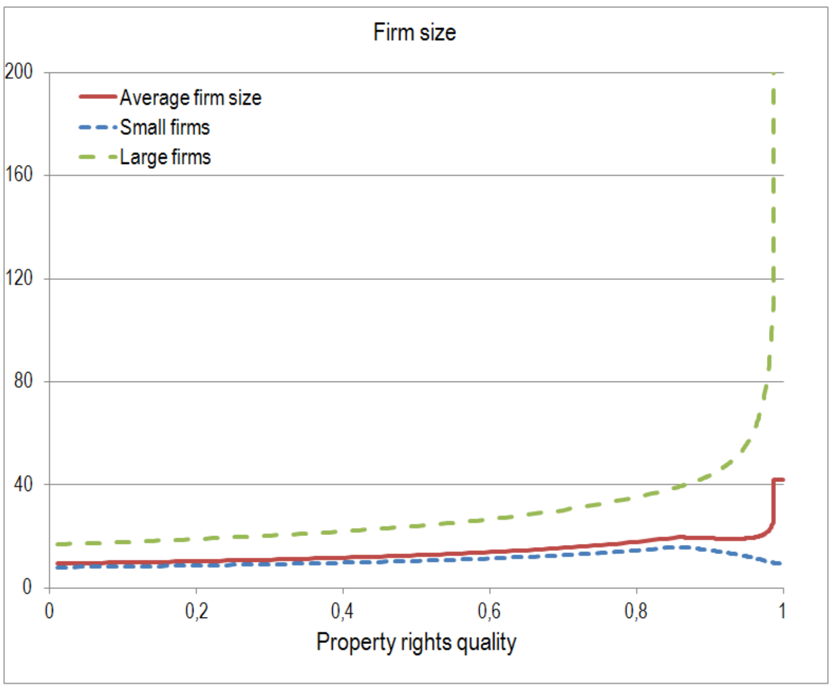

Figure 5: Impact on firm size

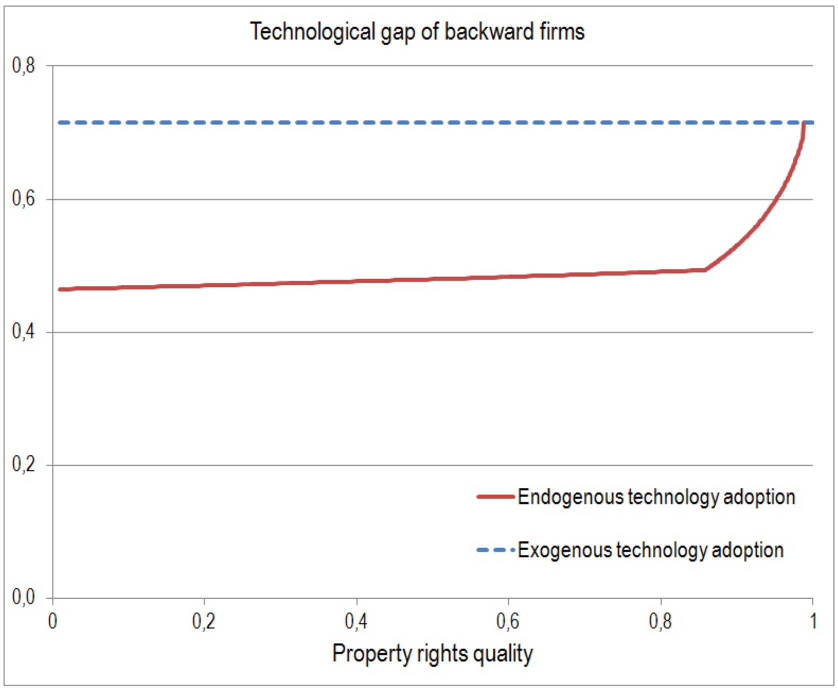

Figure 6: Impact on technological gap between backward and innovative firms

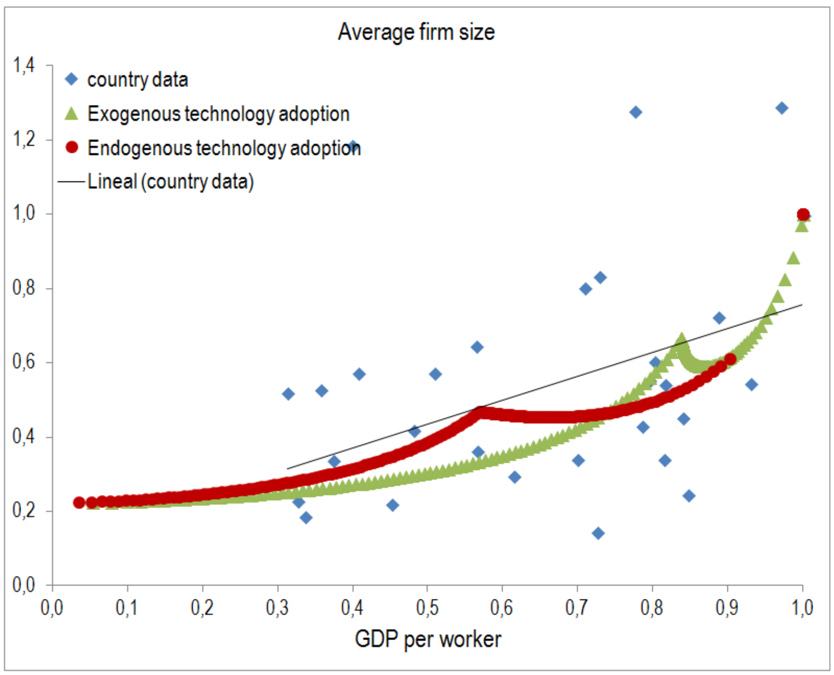

Figure 7: Model vs. data: Impact of property rights (proxied by firm size) and adverse selection on output

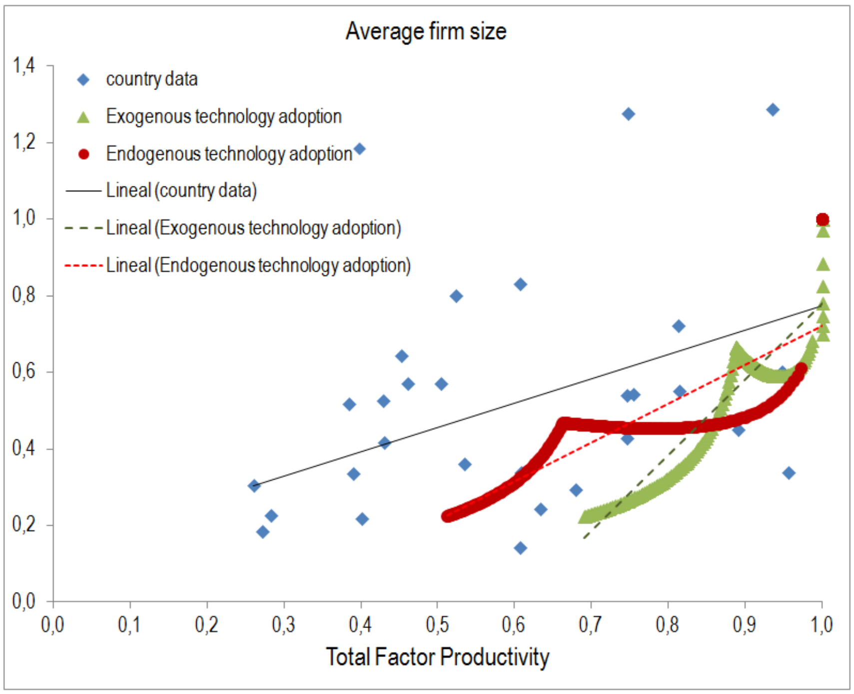

Figure 8: Model vs. data: Impact of property rights (proxied by firm size) and adverse selection on TFP